Mathematics Framework
Chapter 9: Structuring School Experiences for Equity and Engagement

Second Field Review Draft

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Introduction

California schools are tasked with building on widely varying student backgrounds to meet many different needs in mathematics learning. As the student body in most schools has become increasingly diverse in terms of language, culture, socio-economic status, past experience, and interests, it becomes important to consider carefully the best ways to enable all students to excel in mathematics. All students are different; this is a fact to be celebrated. Teaching would not be as rewarding and interesting if all students thought and worked in the same ways. Teachers of mathematics are accustomed to classrooms of students who can offer different ideas and strategies, with some having prior exposure to particular mathematical concepts and some not. Additionally, some students grasp certain ideas more quickly while others appreciate more time to think about those ideas and engage with them more fully. These differences do not indicate different amounts of mathematics potential.

Mathematicians, some of the highest-level achievers in mathematics, often share that they think slowly and deeply. Laurent Schwartz, who won the Fields Medal in mathematics, reflected on his school days with these words:

> I was always deeply uncertain about my own intellectual capacity; I thought I was unintelligent. And it is true that I was, and still am, rather slow. I need time to seize things because I always need to understand them fully. Towards the end of the eleventh grade, I secretly thought of myself as stupid. I worried about this for a long time.
>
> I'm still just as slow. (...)At the end of the eleventh grade, I took the measure of the situation, and came to the conclusion that rapidity doesn't have a precise relation to intelligence. What is important is to deeply understand things and their relations to each other. This is where intelligence lies. The fact of being quick or slow isn't really relevant. (Schwartz, 2001).

Despite the fact that many high-level mathematics users are slow, deep thinkers, it has long been a practice in mathematics education to value speedy thinking, and fast memorization of facts. However, deep understanding should be the primary goal of classrooms—it is from this deep understanding that applications are possible, discoveries are made, and future learning can take flight.

Some people believe that the goal of mathematics learning is to push students ahead as fast as possible, often because they themselves learned mathematics as a series of procedures to memorize. If mathematics is reduced to a subject of memorization, then students who can memorize faster can race through content to higher levels more quickly. As explained in the previous chapter, mathematics experts and leading institutions of higher education have concluded that racing through mathematics without deep understanding is misguided, as it does not develop the mathematical foundation that is required for ongoing progress in quantitative fields.

Students arrive in classrooms with varying mathematical preparation, but it is important to recognize that no student is fixed in their mathematical ability, and all students are on a growth journey as they learn mathematics. For many years it has been assumed that people are either born with a “math brain” or not. This idea has been widely disproved by neuroscience showing that students are forming, connecting, and strengthening mathematical brain pathways each time they learn (Boaler, 2019; Doidge, 2007;

Maguire et al., 2006). This does not mean that all people are born with the same brain; it does mean that abilities grow through the many opportunities students receive for brain development.

In addition, research has found that many characteristics typically used to identify people as “not a math person” have been based on gender, race, and language stereotypes (Chestnut, Lei, Leslie, and Cimpian, 2018; Fennema, Peterson, Carpenter, and Lubinski, 1990; Del Pinal, Madva, and Reuter, 2017; Elmore and Luna-Lucero, 2017; Tiedemann, 2000). Thus, efforts to sort students by perceived “ability,” especially at younger ages, have often served to consign students to “lower” tracks and less rich mathematical experiences in ways that reflect cultural stereotypes and prior exposure to meaningful mathematics.

Many studies show that these perceived differences in ability can be changed by interventions (Kwon et al., 2021; Boaler et al., 2018; Frontiers et al., 2007; Moses and Cobb, 2002). A number of cases of student achievement demonstrate that mathematical excellence can develop or reveal itself at any life stage. Consider, for example, Nicholas Letchford, who started school labeled as having a low IQ and significant special educational needs and went on to graduate from Oxford University with a doctorate in applied mathematics (Letchford, 2018).

While this framework recognizes that some students are born with learning challenges and some are born with learning advantages, and that students experience different opportunities before they arrive at school, it also recognizes that all students are capable of strong achievement gains, with effective teaching and mindset messages. Attaching labels to students is unproductive, as such labels lead to fixed ideas about ability. There persists a mentality that some people are “bad in math” (or otherwise do not belong), and this mentality pervades many sources and at many levels.

The design of pathways that exclude most students from studying the higher-level courses valued by colleges draws upon the incorrect idea that some students cannot learn higher level mathematics. The number of courses generally required before calculus has often caused districts to provide an advanced and accelerated track for some students and a separate track that filters most students out of high-level mathematics from a young age. Many districts in California move students into different pathways at the end of third grade, restricting some from reaching higher level mathematics. Often, students cannot easily change pathways, as they are taught different content. Other districts that sort students into different tracks in middle school use test data from fourth or fifth grade to determine students' mathematical futures. This unnecessarily limits their future attainment and is not a justifiable practice, as educators cannot predict what a student can do in their later years from their elementary school achievement or English language facility at that time.

While acceleration is not in itself a problem when students are prepared for the content they encounter, approaches to tracking that determine early on that many other students will not have the opportunity to take challenging mathematics is a problem, and has contributed to the lack of students who are qualified for STEM futures. In short, the goal should be to develop approaches that enable as much access to higher level mathematics as possible for as many students as possible, developing potential along the way. This chapter provides pathways to this goal for districts to consider.

This framework proposes grouping systems and other supports that keep higher level pathways open to more students for a longer time—while enabling exceptional students to move at a faster pace through courses as needed. The framework recognizes the diversity of student achievement and sets out ways to teach students so that all students receive appropriate support and challenge. It also recognizes the importance of providing all students with challenging work, not leaving any students bored or working at lower levels by requiring that all students stay together or learn the same content at the same pace. High-achieving students may be challenged in a variety of ways, including by acceleration through a course pathway, by engaging in additional mathematics learning opportunities in additional courses or extracurricular challenges, and/or through engagement in ambitious inquiries in any given course. When acceleration occurs, it should be in the context of enabling access for students who are clearly ready for more challenging content at a moment in time, rather than in the context of reducing the opportunities for other young people to access challenging content from which they could benefit.

Instead of reserving high-level content to small numbers of students and denying it to most others through decisions made early in their school careers, this framework recommends approaches that can offer high level mathematics in a variety of ways to all students. This chapter describes methods of teaching that enable all students to be appropriately challenged, without labels, without requiring that all students work on the same mathematics, and without blunt methods of tracking that filter many students out of STEM pathways. The goal is to expand access to rigorous mathematics for all students, so that each experiences the joy and excitement of well-taught mathematics in ways that stimulate their learning and engagement in their own mathematically-rich educational and career pathway.

A History of Tracking in Mathematics

The fixed mindset about mathematics ability—that one either has a brain for mathematics or does not—which is reflected in systems that sort students into different learning opportunities early, helps to explain the exclusionary role that mathematics plays in students' opportunities, and that has led to widespread inequities in the discipline of mathematics. Some of these include

- students who do not quickly and accurately perform rote procedures are discouraged from persisting in mathematically-oriented studies;
- students who are learning the English language are often deemed incapable of handling, and denied access to, grade-level authentic mathematics (Thompson, 2017); and
- students with identified learning differences that affect performance on computational tasks are denied access to richer mathematics, even when the learning differences might not affect other mathematical domains (Lambert, 2018).

Mathematics is the most tracked subject in US schools: approximately three quarters of US grade eight students are tracked in mathematics, a proportion that has not changed in many years (Loveless, 2013). For many, this tracking begins in the early years of elementary school or at the beginning of middle school in fifth or sixth grade. Tracking, which is the term for the school-level practice of assigning students to different courses of study that determine their options over many years of education, reflects a long history of inequality.

Students of color, recent immigrants, and those from low-income families have been routinely “tracked down” into less challenging, rote-oriented coursework that is also generally less well-taught, in large part because these classes are often assigned to the least experienced and expert teachers. Tracking of this sort has been frequently critiqued because it depresses the achievement of students who are in the lower track and because access to higher tracks is often rationed on the basis of criteria that do not predict success in the more ambitious curriculum (e.g., Darling Hammond, 2001; Callahan, Humphries, and Buontempo, 2020; Boaler and Staples, 2008; Boaler, 2014, Guyon, Maurin, and McNally, 2011; Oakes, 2005).

A meta-analysis of 15 studies on tracking, conducted in and outside the US, indicated that students taught in non-tracked groups that offer a more ambitious curriculum tend to have higher achievement overall. This overall improvement is attributed to significant increases for low and middle achievers and no change for high achievers, who, the studies typically found, achieve equally well (and sometimes a bit better) in non-tracked systems (Rui, 2009). Another review of international evidence about tracking found that, while most Organisation for Economic Co-operation and Development (OECD) countries do not track students until ninth or tenth grade, those that track students earlier increase inequality in learning significantly (Woessmann, 2009). The author concludes that “Early tracking leads to a systematic increase in inequality of student performance between the end of the primary and the end of lower-secondary school (29); furthermore, a country’s “performance level [tends] to be lowered rather than raised by early tracking” (30). Studies in Germany (Matthewes, 2020) and the US (Burris, Heubert, and Levin, 2006) have found positive outcomes for achievement and longer-term academic success from keeping students in heterogeneous groups focused on higher-level content through middle school. Some studies have also shown that high-achieving students are advantaged when they are given opportunities to extend work and discuss mathematical connections in non-tracked groups (Boaler and Foster, 2021; Boaler and Staples, 2008; Sahlberg 2021).

As noted earlier, most OECD countries do not differentiate curriculum or track students until the age of 15 or 16 (ninth or tenth grade) (Woessmann, 2009). Although no country's approach directly translates to the context of another, as noted in Chapter 8 there are common curricular approaches that resemble those suggested in this framework taught in non-tracked classes across many of the highest-achieving nations in mathematics, including Japan, Korea, Estonia, and Finland.

Teaching approaches that focus on big ideas and connections are also those that teach mathematics that is close to the nature of the discipline, that are valued at the university level, and that allows students to take work to very high levels and pursue mathematical topics that might otherwise not be met for years ahead. Fortunately, the goals of equity and of high mathematics achievement are not in tension (Matthewes, 2020), as approaches that enable students to get to the highest level of achievement are also those that work to support all students, as described in this chapter.

As a result of the documented issues with early tracking, the National Council of Teachers of Mathematics (NCTM) strongly advocates for creating a system of middle school mathematics courses that will "dismantle inequitable structures, including tracking teachers as well as the practice of ability grouping and tracking students into qualitatively different courses" (NCTM, 2020). NCTM has made clear that if the US is to regain its lost ground in mathematics, districts and schools must confront the structural inequities of tracking and ability grouping that restrict most students from accessing higher level mathematics, and to strengthen their efforts to support all students in learning a common, rigorous curriculum.

While early tracking of students into low-level courses has been problematic, there is evidence that thoughtful grouping of students to ensure they receive high-quality instruction geared to their needs at a moment in time can be helpful. This includes students who need to fill in gaps in their prior learning and high-achieving students who are ready to be more intensely challenged. It is also true that teaching heterogeneous classes requires greater skill for differentiating supports than teaching in classes where the range of performance may be narrower, and should be accompanied by high-quality professional development to enable success.

The research on tracking has not produced a single answer about the best ways to group students, in large part because tracked and non-tracked classes often have different curriculum, class size, teaching goals, and teachers (Steel, 2005). And while “tracking down” has typically been found to undermine achievement for those assigned to lower level courses, some studies have found that “tracking up” into more challenging classes has particular benefits for students of color (see, for example, Card and Giuliano, 2016, who also found that high-achieving students of color are typically overlooked for these opportunities).What is clear is that when tracking systems filter students out of higher-level pathways from an early age, they learn less overall, and they do not get access to STEM careers. Any system that includes acceleration options for some students should do so without excluding most students from reaching higher level mathematics by the end of high school. Chapter 8 suggested that it may be fruitful to consider how the pathway to pre-calculus or calculus and other advanced mathematics courses may be made more efficient in high school, and how students might get access to supplemental coursework, so as to allow greater access to advanced mathematics for more students without shortchanging the foundational learning they need in order to succeed.

This chapter discusses various options for grouping and instructing students that include more flexibility, fair placement, and opportunity for more students. The goal of this framework is to ensure that as many students as possible excel at mathematics through the kinds of curriculum access and teaching methods that will support their success and carry them into opportunities that open up both the beauty of mathematics and a wide range of quantitatively-grounded careers to them.

The remainder of this chapter sets out the different ways students may be challenged and supported in mathematics classes with examples of districts and schools that have enacted systems of grouping that support a wider range of students in accessing higher level content. If the goal is to open mathematics pathways to more students and give greater challenge to high achieving students to develop broader proficiency and long-term interest in quantitative fields, then this framework recommends reshaping the content that is offered to students—the way it is taught, and the organization of students learning the content—in the following ways.

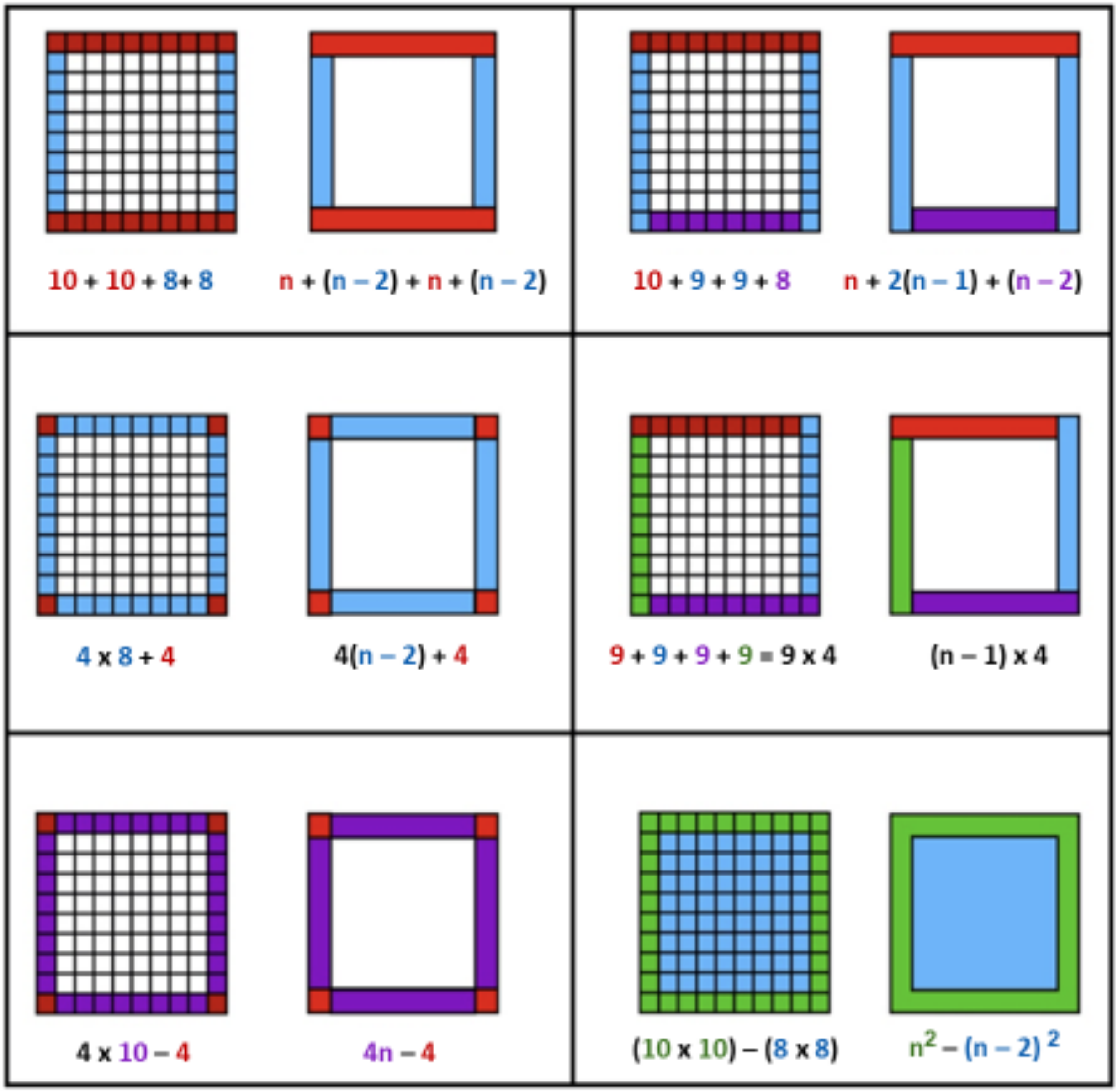
Teaching Multidimensional Mathematics through Big Ideas and Connections

A number of schools, districts, and educational systems have worked to open pathways to high achievement to significantly more students by eliminating low level classes in mathematics and teaching a broader and more multi-dimensional mathematics to all students. Such an approach allows students to explore questions of interest and work on mathematics at different levels. Instead of teaching through narrow questions that are only accessible to some students, with some students not gaining access and others being bored and unchallenged, students are taught mathematics through more open tasks that they can approach in different ways and take to different levels.

For example, in a typical algebra classroom students might be asked to simplify these expressions:

1. $n + (n +2) + n + (n -2)$
2. $4 (n-2) + 4$
3. $n + 2(n -1) + (n-2)$
4. $4(n-1)$
5. $n^2 - (n-2)^2$

In a classroom focused on big ideas and connections, the teacher may choose generalizing as a big idea, and introduce the idea through the “border problem” as explained in chapter 7. In this approach students consider the tiles on a border of different sized squares, eventually describing the border size with words, and then algebraically, and then forming equivalent algebraic expressions. This is a more open task than the first, as it allows students to explore and make connections in multiple ways. It is also a task with a low floor and high ceiling—all students can visualize borders of squares—and higher achieving students can extend the problem to borders of different shapes. The task also encourages the principles of UDL—students can engage with it in different ways—with visuals, words, numbers, and discussion. This creates a multidimensional mathematics experience for students, and the task leads to a deep understanding of generalization and equivalent expressions. The first task is one dimensional—students simplify expressions. The border task is multidimensional as students engage in many dimensions of mathematics—generalizing, visualizing and drawing, communicating, connecting words, expressions and visuals. Such tasks take longer than a narrow question involving equivalent expressions but research has shown that a teaching approach geared to big ideas, with fewer tasks that are deeper and longer, not only engages all students—whatever their prior achievement—but also increases understanding for all students, including the highest achievers (see also Nasir et al., 2014; Boaler and Staples, 2008).



Link to long description

There have been numerous research studies showing the effectiveness of approaches that focus on big ideas, and multidimensional mathematics, with students from different achievement levels working together. In a de-tracking initiative, a suburban New York school district stopped teaching “regular” or “advanced” classes in middle school, and instead provided all students with content previously labeled as “advanced.” Researchers followed students in six cohorts over six years. In the first three years the cohorts worked in tracks, for the next three years the cohorts worked in heterogeneous classes using the “advanced” curriculum, which consisted of sixth, seventh, and eighth grade coursework taken in grades six and seven, followed by the first course in an integrated mathematics sequence incorporating algebra concepts (entitled Sequential Mathematics I) in eighth grade. The researchers found that the students who learned in the heterogeneous classes took more advanced math, enjoyed math more, and passed the state Regents test in New York a year earlier than students in traditional tracks. Further, researchers showed that the advantages occurred across the achievement spectrum for low and high achieving students (Burris, Heubert, and Levin, 2006).

Another study describes a county-wide approach in which fifth grade teachers across several districts in California’s Central Valley were taught to teach multidimensional mathematics. Within one year the students significantly increased their mathematics achievement on CAASPP tests—particularly girls, language learners and economically disadvantaged students (Anderson et al., 2018). Boaler and Foster (2021) describe the change in achievement that resulted when teachers in eight districts in Northern California were given professional development that helped them de-track middle school classes and teach broader and deeper mathematics. Student achievement in these districts was compared with that in districts who continued to teach students in tracked groups with a more narrow mathematics focus. In the non-tracked districts, 15 percent more of the students achieved proficiency in the CAASPP assessments and 20 percent more students in the more conceptual MARS assessments (Boaler and Foster, 2021). In a second study, comparisons were made between students working in tracked groups and the same districts one year after significant de-tracking with the use of a more conceptual curriculum. After a large number of districts de-tracked mathematics in middle school, student achievement increased significantly across the achievement range, as shown in figure 9.1.

Figure 9.1: Student achievement when students were arranged into regular or advanced classes in eighth grade

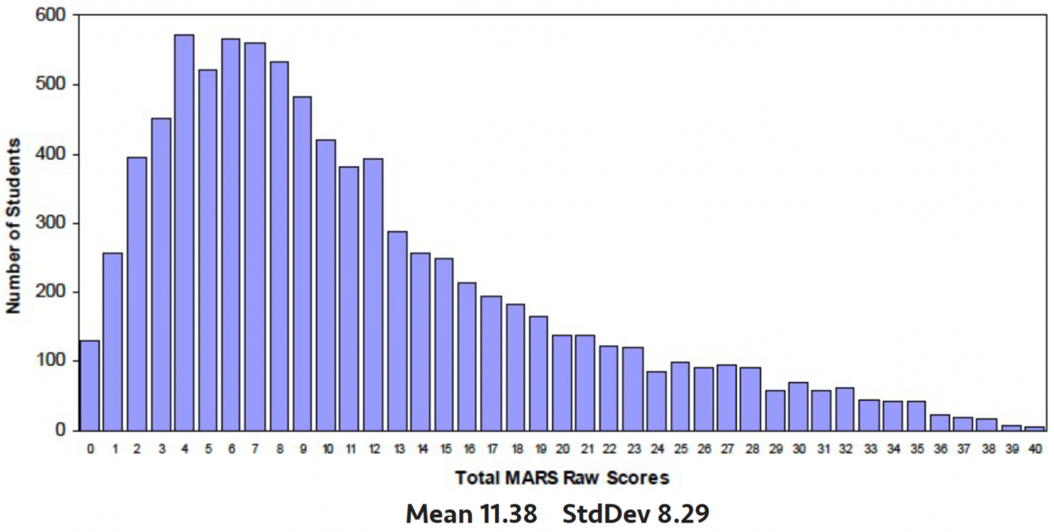
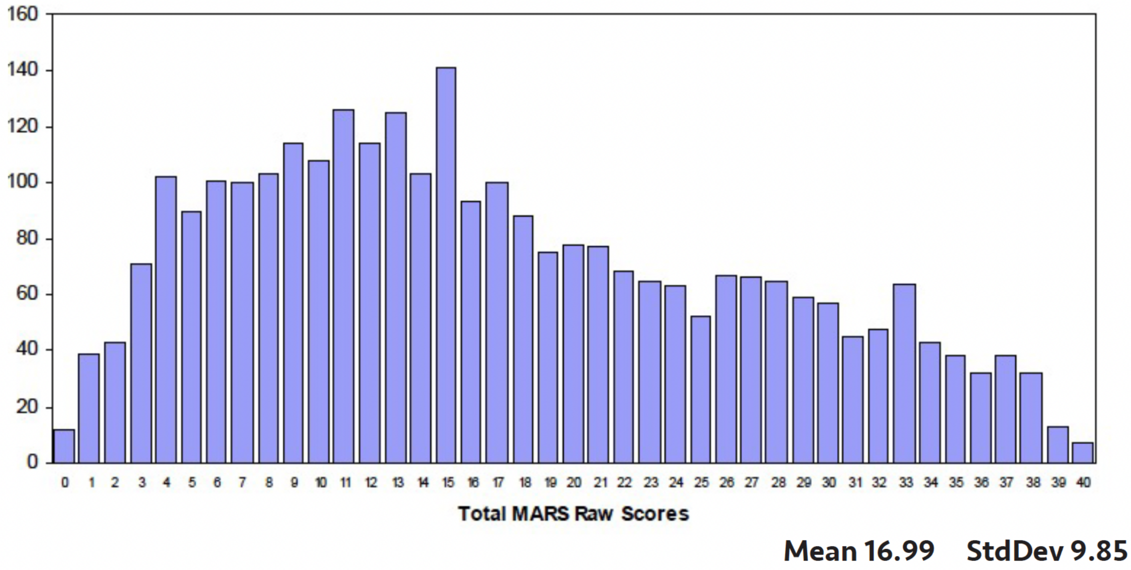


Figure 9.2: Student achievement when the majority of students took CC Math 8



These distributions show that student achievement increased across the range when students were taught a more conceptual curriculum in de-tracked groups, producing significantly more high achieving students. The score gain of 5.61 on the assessments (0.68 standard deviations), is equivalent to 2.03 years of middle school growth.

Two longitudinal studies, one in the US and one in the UK, followed students over four and three years, respectively, from the ages of 11 to 18. The studies aimed to consider the impact of tracking, curriculum choices, and teaching. In both studies, students in schools using heterogeneous groups as part of equitable initiatives that gave students a multidimensional mathematics experience achieved at significantly higher levels than students in schools employing traditional teaching and tracking. The schools achieved success with heterogeneous grouping by using low floor high ceiling tasks that all students could access and that students could take to very high levels (also see Chapter 2). This success held across different countries, cultures, and schools (Boaler, 2011, 2015, 2016; Boaler and Staples, 2008). A follow-up study revealed that students who attended the heterogeneously grouped school, which emphasized problem solving over procedures, were in more professional jobs and had longer term success as adults (Boaler and Selling, 2017).

An important resource for districts and schools that choose to offer higher level mathematics to all students are the textbooks and instructional resources that teach to big ideas and connections. Textbooks that share deep mathematics tasks and can be worked on across a sequence of multi-day investigations are appropriate, as opposed to textbooks that offer short, closed questions, with limited interest or appeal to many students. In high school, truly integrated content provides greater opportunities for broad and deep tasks that give appropriate challenge for all students. In studies of these curricula in use in urban, suburban, and rural districts, including those in California, students have achieved at significantly higher levels on tests of problem solving, conceptual understanding, and applied mathematics, and have enrolled in significantly more high school mathematics courses (Core-Plus Mathematics, n.d.).

Since most teachers did not learn mathematics in this way, substantial commitment on the part of districts and counties is required to support teachers in learning and implementing instruction that keeps all students challenged and engaged. Chapter 10 describes a multitude of opportunities for teachers to learn this fulfilling approach, through district/county/state professional learning and online workshops. When teachers learn to teach using low floor, high ceiling tasks they report greater student engagement, with students of different achievement levels being productively challenged in different ways, and higher teacher satisfaction (Boaler, 2019).

Personalized Learning

Another teaching method for attending to the different achievement levels of students is to provide personalized pathways, so that any advancement comes from students demonstrating readiness through their work, rather than as a result of group-based, long-term tracking decisions made by schools or districts that predetermine how students will be processed through standardized coursework at a standardized pace. This can be achieved through teachers allowing students to work through courses at different paces, illustrated by the “personalized by teachers” vignette below, or with the support of computerized systems, as illustrated in the “personalized learning” vignette in chapter 2. New innovative learning models, that include methods of assessing and placing students using technology, are a step forward from methods of tracking that often keep students in the same place and have students in the same classroom working through the same work at the same pace.

Vignette: Personalized by Teachers

A high school mathematics department wanted to tackle the problems of fixed tracking systems by giving students’ choice and allowing their work in different courses to decide which course they took. The teachers made an arrangement for students to take assessments at the end of each course unit and to move at the pace most appropriate to them.

In this team-taught program each student is assigned to a lead teacher, who sets goals for them and tracks their progress. Students meet with their lead teacher each day at the beginning of the period to work on open problems, or number or data talks, as a class. Students then transition to different rooms for each course (e.g., Algebra, Geometry, Algebra 2, or Trigonometry), where they sit in groups and work on the course materials. The teachers for the courses circulate around the room providing small group instruction, asking guiding questions, keeping students on task. When a student finishes a topic, they submit a request to be assessed, which their lead teacher approves after checking that they have completed all of the materials for that topic. Students then take an assessment and, if they have achieved at least 70 percent, they are free to move on to the next topic. If they get below 70 percent, they work with their lead teacher to relearn the materials. Students also have the option to remediate any assessment, regardless of their score, and teachers always take the higher grade. Once students have completed all work from their course they transition directly into the next course. Classes are team taught and multiple courses (such as Algebra, Geometry, Algebra 2, and Trigonometry) run in the same period.

This approach has allowed students to exercise agency and to move ahead whenever they have learned the material for a course. The teachers reflect that

> “Some students, who have always hated math, have grown to love it because they are able to take control of their learning. They move at the pace that is right for them and while it may be slower than in a traditional year-long class, they feel like they are finally learning the material and their assessment scores show that. Other students have embraced the idea that sometimes you need to slow down in order to pick up speed later. When they take time to build that strong foundation they find they can pick up speed later. Other students have set lofty goals for themselves and have a strong desire to complete multiple courses in one year. Given that they are demonstrating mastery on their assessments, we don't believe in holding them back. This is allowing students to have multiple pathways to higher level math courses. They are no longer limited by a placement decision most likely made in sixth grade. Students can still start high school in algebra and get to calculus or beyond if that is their goal.”

By *Personalized Learning (PL)*, this Framework means learning experiences that are customized “…for each student according to his or her unique skills, abilities, preferences, background, and experiences” (Herold, 2019).

The great benefit of personalized systems is that they allow students to work at their own pace, on content that is appropriate for their understanding. In order also to experience the insights of others and engage in joint problem solving, individualized experiences can be combined with opportunities for mathematical collaboration.

Some of the best systems available have worked to combine different experiences—with students spending some time working on a computer and other times working with other students and the teacher on rich mathematics developing conceptual understanding. In one successful teacher-developed approach, students engage in blended, self-paced, mastery-based learning with teacher made videos supplementing in-class problem-solving individually and in collaborative groups, with continual assessment and revision of work moving students toward confidence and competence (Modern Classroom, 2021). A similar model developed by a middle school teacher and now taught in many schools, uses diagnostic assessments to create a tailored set of assignments for each student that the teacher can use in technology-infused mix of direct instruction, collaborative work with peers, and individualized learning (Margolis, 2019). A third approach -- offering multiple strategies that can be used in individual or collaborative study to learn and practice content mapped to standards at each student's level of mastery – has been found to reduce math anxiety and to support greater achievement for students at different initial levels of achievement when used to complement classroom instruction (Murphy et al., 2014). The goal should be to create a personalized learning environment that is focused on rich mathematics, through which students can conduct mathematical investigations and work on big ideas and mathematical connections.

For now, it is crucial that schools and districts considering personalized learning products or programs review them carefully to ensure that they:

- Develop mathematical concepts, problem-solving strategies (including computation), and application in a way that each supports the other.
- Design student activities around big ideas that connect multiple content standards through engagement in the Standards for Mathematical Practice (SMPs) in the context of authentic investigation.
- Emphasize connections between mathematical ideas, strategies, and representations, rather than isolated skills.

- Include collaborative components in student investigations, to build mathematical content and practices that emphasize mathematical communication and discourse.

While research on the effectiveness of many personalized systems is scant (Zhang et al., 2020), emerging technology-based systems offer promise for educators striving to meet individual needs of learners across the achievement spectrum in heterogeneous classrooms (Deunk et al., 2018).

Districts are also increasingly deploying one-on-one or small group tutoring to help students secure skills that they may have missed or not fully mastered. A growing research base shows that specific programs offered by trained tutors with frequent, regularly scheduled sessions can result in very substantial gains in mathematics achievement, allowing students to accelerate their learning and sustain a path to higher level courses (US Department of Education, 2017; Nickow, Oreopoulos, and Quan, 2020). Systematic use of tutoring could reduce the felt need for lower-track classes that derail students from STEM careers at early grades.

Additional Classes

The negative effects of traditional tracking systems become particularly evident when students start high school, as high-level courses such as calculus are often only available to students who have been accelerated in middle school, a problem that is exacerbated when students struggle with any of these classes. While high schools still require more courses than students can take, they should develop ways for students to catch up with content as well as accelerate their learning. For example, schools could offer a summer class before students attend high school for students to strengthen their readiness for the next sequence of courses. For example, the Algebra Project, created by Bob Moses, has designed curriculum used in after school and summer programs, as well as during school-year courses, that enable students both to strengthen their skills and to develop a strong mathematics identity. Similarly, the Calculus Project enables higher achievement for traditionally underrepresented students by working with schools to offer preparatory courses in the summer, as well as after school study groups and tutoring during the school year to support mathematics instruction from grades eight through twelve.

As another way to accomplish this goal, Louisiana implemented a pilot, in which students enrolled in two periods of Algebra I, with the same teacher for both periods, using curriculum that interwove foundational mathematics and algebra content together. The extended time and additional supports for teachers were critical to the success of the project. Academic support courses for high-school mathematics have been shown as effective in a number of studies. (See US Department of Education, 2018.) The support courses are offered to provide additional time for classroom instruction (as in the case of the Louisiana project), homework support, and supplemental assignments, emphasizing study skills and preparation in the core companion courses.

Some districts offer support classes without communicating to students that they are at a lower level by opening the classes to everyone. In one highly successful case, middle schools in a California district offered a class that followed the regular mathematics class and was open to all students (see Boaler, 2016). The content was the same as the class just taught but the extra time allowed students to discuss the ideas more and ask more questions. Many students chose to enroll in the class—high and lower achieving students. It is important that such classes are named positively, not as remedial classes but as additional depth classes.

Additional opportunities may be provided outside of the regular school day. These can give students experience of mathematics that they may not meet in school, and can also offer a more investigative approach. Two highly regarded examples are math circles and Math Olympiad classes. Math circles are communities that offer opportunities for mathematical problem solving—and they are available for students and for teacher professional learning (Math Circle Network, n.d.). Mathematical Olympiads for Elementary and Middle Schools (MOEMS) is a program sharing Math Olympiad opportunities with elementary and middle school students and teachers (Math Olympiads for Elementary and Middle Schools, n.d.). These different opportunities for students to engage in mathematical problem solving outside of regular school hours, are often highly successful as they can help students develop a positive mathematics identity (Langer-Osuna, 2007, 2017) and start a new approach to mathematics.

More Flexible Versions of Student Grouping

A number of California districts are attempting to improve opportunities for all students to excel in mathematics through more innovative approaches to grouping students, that help to ameliorate the negative impacts of tracking. Examples include:

- Moving the beginning of separate course pathways to later grades—e.g., from sixth grade to eighth or ninth grade, or from ninth grade to tenth or eleventh grade. Since student interest and engagement can fluctuate significantly during adolescent years, this enables more students to build a strong mathematics identity before they are assigned to (or choose) a placement. Making placement decisions no earlier than eighth grade gives students four more years than some current practices to discover their interest in mathematics and to demonstrate their engagement and understanding. The best approaches to separate pathways are those that give students and families the choice as to which pathway their student takes.

- Reverse-engineering high school pathways so that all advanced courses (including Advanced Placement courses) are attainable by students beginning with the default course in ninth grade. As discussed in Chapter 8, this is possible because the California Common Core State Standards for Mathematics (CA CCSSM) middle grades expectations are sufficiently rigorous preparation for a four-year high school pathway to include advanced classes including AP Statistics or Calculus.

- Designing high school pathways that begin with a common geometry course in ninth grade. After the introductory geometry year schools are better placed to choose the next course for students, in consultation with families.

- Factoring student interest and desire more heavily into student placement, recognizing that many students blossom when they are offered higher level content and they choose to step up to challenges, especially when they have the support they need to succeed.

- Offering data science as a high school course. Students do not need to have been in an advanced track to take this course. Many students who have developed the idea that mathematics is not for them may be re-energized and re-inspired by access to twenty-first century content that is important for their lives, and that can, with additional course taking, lead to a future in STEM. See chapter 5 for more detail on the possibility of high school courses in data science, recognized as A–G courses by the colleges of California.

This chapter has described some of the problems with traditional methods of early tracking and described alternative approaches to student grouping, especially in elementary and middle school. Districts that choose to continue traditional tracking need to take note of the California Mathematics Placement Act of 2015, which requires that every high school placement policy of a local educational agency meet the following requirements (CDE, 2016):

- Systematically takes multiple objective academic measures of pupil performance into consideration;

- Includes at least one placement checkpoint within the first month of the school year to ensure accurate placement and to permit reevaluation of individual student progress;

- Requires an annual examination of pupil placement data to ensure that students are not held back in a disproportionate manner on the basis of their race, ethnicity, gender, or socioeconomic background;

- Requires a report on the results of the annual examination by the local educational agency to its governing board or body;

- Offers clear and timely recourse for each pupil and his or her parent or legal guardian who questions the student’s placement; and
- For non-unified school districts, addresses the consistency of placement policies between elementary and high school districts.

Discussion and Conclusion

Districts are at liberty to group students as they choose, but for districts wanting to open mathematics pathways to more students and create opportunities for higher achievement, there are many options to consider, as this chapter has described.

In the elementary years, students should experience common mathematics content. Students may work in different ways and at faster or slower rates, but this does not mean that they should be exposed to different content. Many of the teaching approaches and activities described in Chapters 3 and 6 emphasize multidimensional learning over speed and memorization. When mathematics questions are multidimensional and invite students to engage in reasoning, making connections, and seeing and representing ideas in different ways, they can engage all students appropriately. In addition, strategies like tutoring and personalized supplementary programming can help students secure and reinforce skills that allow them to progress successfully in the curriculum.

Middle schools also have an important role to play in ensuring that all students receive well-taught, challenging coursework that does not close off later options. By maintaining rich mathematical content along with strong and supportive teaching, they can make access to higher-level mathematics more likely for a greater share of students. Because many of the topics included in the former Algebra I course are in the CA CCSSM for grade eight, the Mathematics I and Algebra I courses that build on the CA CCSSM for grade eight are more advanced than the previous courses. These typically start in ninth grade with more advanced topics and include more in-depth work with linear functions and exponential functions and relationships, and they go beyond the previous high school standards for statistics. The integrated course pathways that start with

Mathematics I or Mathematics: Investigating and Connecting build directly on the CA CCSSM for grade eight and provide a seamless transition of content through an integrated curriculum. (See Chapter 8 and Appendix A for discussions of various high school course pathways.)

Even when high schools differentiate mathematics course taking options, they can take the lead in opening up opportunity for more students to engage in advanced course-taking by reducing the redundancies in current courses that may unnecessarily slow progress toward the highest-level courses like calculus or statistics. This framework proposes that the state convene experts to evaluate how that might be accomplished. High schools can also provide multiple ways to reach these courses—through block scheduling or supplementary courses, by continuing tutoring opportunities, and/or by offering high level courses, such as data science, that do not need prior acceleration.

By designing curriculum and teaching in ways that invite personalization and by providing open-ended tasks that can be approached in many ways to enable deep learning, teachers can enable more students to tackle ambitious mathematics successfully. Those who are already eager and able mathematicians will be able to excel with a stronger foundation, and they will be joined by more of their peers who have greater opportunities to develop their potential.

Long Description for Chapter 9

Border Problem

Six rectangles include two squares each. Squares include borders comprised of various shadings. Rectangle one includes two squares shaded to indicate 10 + 10 + 8 + 8 and n + (n – 2) + n + n (n – 2). Rectangle two includes two squares shaded 10 + 9 + 9 + 8 and n + 2(n – 1) + (n – 2). Rectangle three includes two squares shaded 4 x 8 + 4 and 4(n – 2) + 4. Rectangle five includes two squares shaded 9 + 9 + 9 + 9 = 9 x 4 and (n – 1) x 4. Rectangle five includes two squares shaded 4 x 10 – 4 and 4n – 4. Rectangle six includes two squares shaded (10 x 10) – (8 x 8) and n squared - (n – 2) squared. Return to graphic.

California Department of Education, March 2022