Mathematics Framework
Chapter 9: Supporting Equitable and Engaging Mathematics Instruction

First Field Review Draft

Note to reader: The use of the non-binary, singular pronouns *they*, *them*, *their*, *theirs*, *themself*, and *themselves* in this framework is intentional.

Introduction

A broad system of support is needed to ensure that all students have access to mathematics instruction that reflects authentic contexts and real-world problems, is rich with connections between mathematical ideas and with students' lives, and builds over time. As students learn and process mathematics, their teachers learn the effects of

their practice and refine their teaching; together these processes form the core learning environment for mathematics. So, how can teachers be best supported in creating equitable and engaged mathematics learning environments for their students? Administrators and teacher leaders, such as coaches and teachers on special assignment, provide the initial, programmatic layers of support, while parents, counselors, and community members co-create an interconnected system that supports children and adolescents as they learn. This chapter presents guidance designed to build for teachers an effective system of support as they facilitate learning for their students.

Start callout box

Authentic: An authentic problem, activity, or context is one in which students investigate or struggle with situations or questions about which they actually wonder. Lesson design should be built to elicit that wondering. In contrast, an activity is inauthentic if students recognize it as a straightforward practice of recently-learned techniques or procedures, including the repackaging of standard exercises in forced "real-world" contexts. Mathematical patterns and puzzles can be more authentic than such "real-world" settings.

End callout box

It is crucial that anyone making professional learning plans for mathematics teachers understand the vision of mathematics teaching and learning described in this framework. As described in Chapter 2, the goal "is for students to view mathematics as a vibrant, inter-connected, beautiful, relevant, and creative set of ideas." Chapter 2 develops five central themes of instruction that develop this view of mathematics.

1. Plan teaching around big ideas.
2. Use open, engaging tasks.
3. Teach toward justice.

4. Invite student questions and conjectures.
5. Center reasoning and justification.

In addition, Chapter 2 presents a framework drawn from Darling (2019) that is important for supporting linguistically and culturally diverse English learners, as well as other students:

1. Take an asset approach and recognize multilingualism as a power.
2. Include group work (strategically grouping for language development).
3. Make work visual (include graphic organizers, visual examples, encourage visual communication).
4. Build on students' lived experiences and cultures (allow native language use).
5. Scaffold learning and language development (sentence frames, sentence starters).
6. Give opportunities for pre-learning (giving students opportunities to learn some prerequisite material ahead of time).

Professional learning experiences for teachers, teacher leaders, and administrators must be designed to support instruction that implements these themes.

The framework's progression chapters (Chapters 3–5) illustrate instruction through the development of major mathematical strands—mathematical practices and content—across the full TK–12 grade continuum, and grade band chapters (Chapters 6–8) further detail ways educators can maintain a focus on big ideas and implement instruction in developmentally-appropriate ways. Classroom activities designed around big ideas will typically pair one or more Content Connections (CC; broad categories of mathematical content) and one or more Standards for Mathematical Practice (SMP) with a Driver of Investigation (DI; purposes for pursuing mathematical learning). Because instruction is so tied to these three dimensions, these three dimensions should also play a major role in the design of professional learning.

Drivers of Investigation

- DI.1: Making Sense of the World (Understand and Explain)
- DI.2: Predicting What Could Happen (Predict)
- DI.3: Impacting the Future (Affect)

Content Connections

- CC1: Communicating Stories with Data
- CC2: Exploring Changing Quantities
- CC3: Taking Wholes Apart, Putting Parts Together
- CC4: Discovering Shape and Space

Standards for Mathematical Practice

- SMP.1: Make sense of problems and persevere in solving them
- SMP.2: Reason abstractly and quantitatively
- SMP.3: Construct viable arguments and critique the reasoning of others
- SMP.4: Model with mathematics
- SMP.5: Use appropriate tools strategically
- SMP.6: Attend to precision
- SMP.7: Look for and make use of structure
- SMP.8: Look for and express regularity in repeated reasoning

In an attempt to provide some consistency across subjects for those seeking to create opportunities for professional learning, this chapter of the Framework mirrors in structure Chapter 12 (Implementing High-Quality Science Instruction: Professional Learning, Leadership, and Supports) of the 2016 *California Science Framework* (https://www.cde.ca.gov/ci/sc/cf/cascienceframework2016.asp), and echoes many of its recommendations for supporting quality instruction.

Collaborative Systems of Learning and Support

Teachers perform incredibly complex work that relies on thousands of instructional decisions every day (Ball, 2018): in understanding their students' thinking, choosing tasks, deciding which questions to pose in discussion, selecting which (and whose) lines of inquiry to pursue with the class, and ensuring that all students have their authentic and culturally relevant contexts and tasks represented. When stakeholders and influencers outside of the classroom are not aligned, this work of teaching is made even more difficult, and instructional practice changes little.

The California Common Core State Standards in Mathematics (CA CCSSM) were adopted by the State Board of Education in 2010. While the standards implementation has led to significant change, the iterative nature of teaching means that improvement is ongoing. The continuous improvement of mathematics teaching and learning requires the aligned efforts of many stakeholder communities (adapted from the *California Science Framework*, 2016):

- Teachers and teacher leaders prepared to engage in student-centered teaching that engages students in equity-oriented learning through authentic tasks and contexts that are relevant to those students based on their choices, interests, and aspirations
- School, district, and county office administrators who are knowledgeable and supportive of the changes demanded by the CA CCSSM and this framework
- Afterschool, early childhood, and other expanded learning opportunities aligned with and supportive of authentic mathematics learning that include collaborative and coherent efforts between teachers and other education support professionals
- College and university faculty involved in and advocating for high-quality mathematics instruction and preparation of future teachers

- Community members and parents, guardians, and families who understand the reasons for and are supportive of engaging and equitable approaches to mathematics teaching and learning
- Formal and informal learning environments, including museums, libraries, science centers and other venues that are fully committed to supporting CA CCSSM

Effective progress takes place within these communities when it is aligned with an ongoing cycle of implementation, reflection, and improvement of practice (Little, 2006; Penuel, Harris, and Debarger, 2014; Fixsen, Naoom, Blase, Friedman, and Wallace, 2005; Fixsen and Blase, 2009). The vision is for teachers and other educational stakeholders to engage in a learning community that has the same characteristics—respect, commitment, intellectual engagement, and motivation toward continuous improvement—that all educators hope to create for students in California classrooms.

Ermeling and Gallimore (2013) present models of implementation that have been embedded in school learning communities across 40 districts. These models focus on addressing learning needs common to the members of the community; analysis of evidence is used to drive planning, decision making, and critical questioning of practices. To be effective, the learning community must operate in an environment of collaboration and trust among teachers and school leaders, each of whom recognize that improvement requires time, resources, continuous support, and an appreciation of risk-taking as new instructional approaches are implemented.

An environment that realizes these improvement efforts in mathematics teaching and learning should focus on the sustainability of the instructional practices and education programs—and the sustainability of the professional learning cycle itself—by fostering a collaborative school culture that engages educators, administrators, students, parents, guardians, families, education professionals, and community members (Fixsen & Blase,

2009). Establishing culture allows all stakeholders to understand themselves as advocates and supporters in the effort to improve students' experience and achievement in mathematics.

Finally, the 2014 *California ELA/ELD Framework* (https://www.cde.ca.gov/ci/rl/cf/elaeldfrmwrksbeadopted.asp) calls on teachers and educational leaders to examine personal beliefs and attitudes toward students and their families; the call also certainly applies to teachers across contents, including mathematics instruction. Explicit reflection helps educators approach all students with a growth mindset disposition that both values the cultural resources and linguistic assets students bring to the mathematics classroom and supports them to use these resources while expanding and adding new perspectives and ways of appropriating and using mathematics. Put simply, teachers' beliefs about their students significantly affect those students' motivation, experience, and achievement (Stipek, Givvin, Salmon, & MacGyvers, 2001; Heyder, Weidinger, Cimpian, & Steinmayr, 2020).

As mathematics teaching and learning are complex endeavors (Russ, Sherin, & Sherin, 2016), the complexity of teaching will be a recurring theme throughout this chapter. Indeed, even defining what is meant by improvement of teaching practice involves connected changes in general pedagogy, mathematics pedagogical content knowledge ("ways of representing and formulating the subject that make it comprehensible to others" [Shulman, 1986]), and mathematical knowledge for teaching ("the mathematical knowledge needed to carry out the work of teaching mathematics" [Ball, Thames, & Phelps, 2008]).

Professional Learning for Equity and Engagement: Critical Content

Mathematics education has a long history of inequitable access to rich learning (see Chapter 1 and Chapter 2 for more discussion of this topic). It is incumbent on all in education to seek or create professional learning experiences that are designed to help

teachers challenge and overcome the legacy practices that continue to perpetuate these inequities in access and attainment. Even when professional learning is designed with a different primary focus (mathematical practices, particular instructional routines, or teaching from big ideas, for instance), the implementation of these ideas should reflect culturally relevant and sustaining ways, and include awareness of and attention to the impacts of unconscious bias on students' experiences in the mathematics classroom.

More importantly, the field should prioritize professional learning opportunities that focus primarily on equity in mathematics education; equity cannot be an afterthought to more traditional mathematics content-centered offerings that do nothing to address the fact that "Black, Latinx, Indigenous, women, and poor students, have experienced long histories of underrepresentation in mathematics and mathematics-related domains" (Martin, 2019; see also Martin, Anderson, & Shah, 2017). Inequities caused by systemic issues means that a "culture of exclusion" persists *even in equity-oriented teaching* (Louie, 2017). Many of the stories that we use to define mathematics, and to talk about who does or is good at mathematics, are highly racialized and English language-centric, and are experienced that way by students (Lue & Turner, 2020). This means students' mathematics identities are shaped in part by a culture of societal and institutionalized racism. Professional learning in mathematics can respond to these realities and aim for more than incremental change (which does little to change the framing narratives that drive inequities).

A Pathway to Equitable Math Instruction (Education Trust West, 2020) is a guide to building equity in mathematics teaching (see https://equitablemath.org). It was developed by California educators through a comprehensive partnership of mathematics education and equity organizations. While written specifically to address grades 6–8, the broad outline of the path that is outlined is applicable to all grades:

- Stride 1: Dismantling Racism in Mathematics Instruction: Exercises for educators to reflect on their own biases to transform their instructional practice
- Stride 2: Fostering Deep Understanding: Methods for deepening content understanding and relevance through crafted mathematics discussions
- Stride 3: Creating Conditions to Thrive: Environments and practices that support students' social, emotional and academic development
- Stride 4: Connecting Critical Intersections: The interconnectedness of English language learning and the development of mathematical thinking
- Stride 5: Sustaining Equitable Practice: Coaching structures that support mathematics educators in their ongoing centering of equity principles

The table below (adapted from the 2014 *ELA/ELD Framework*) outlines critical content for professional learning. Due to the inherent complexity of teaching, there is a risk of trying to do everything at once; it is important to design opportunities around a manageable subset of these foci.

Critical Content for Professional Learning in Mathematics Education

Establishing a Vision for California's Students

- Develop the readiness for college, careers, and civic life
- Attain the capacities of numerate individuals
- Become broadly literate in quantitative subjects
- Acquire the skills for living and learning in the 21st century

Understanding the Standards

- CA CCSSM Mathematical Practice Standards
- CA CCSSM Mathematics Content Standards
- ELA/ELD Standards as implemented in Mathematics Classes
- Implementing science, history/social studies, career and technical education, and other standards in tandem with mathematics

Establishing the Context for Learning

- Integrating the curricula
- Motivating and engaging learners
- Teaching from Big Ideas, not individual standards
- Respecting learners, and the cultural and linguistic assets they bring
- Ensuring intellectual challenge

Enacting the Key Themes of Mathematics Instruction

- Mathematics as tools for solving authentic problems in authentic contexts
- Meaning making
- Mathematical practices
- Language development
- Effective expression
- Content knowledge

Addressing the Needs of Diverse Learners

- Comprehensive English language development:
  integrated and designated ELD
- Additive approaches to language and mathematics development
- Meeting the needs of
  students with disabilities and students experiencing difficulty
  Meeting the needs of advanced learners and other populations

Exploring Approaches to Teaching and Learning

- Teaching through investigation
- Models of instruction
- Culturally and linguistically responsive teaching
- Supporting biliteracy and multilingualism
- Supporting students strategically (including UDL and MTSS)

Sharing the Responsibility

- Collaborating within and across grades, departments, and disciplines
- Promoting teacher leadership
- Partnering with community groups and higher education
- Collaborating with parents

Evaluating Teaching and Learning

- Types and methods of assessment (formative, summative, rubrics, portfolios, diagnostic)
- Cycles of assessment (short, medium, long)
- Student involvement in assessment
- Appropriate preparation for state assessments

Integrating 21st Century Learning

- Critical thinking skills
- Creativity and innovation skills
- Communication and collaboration skills
- Global awareness and competence
- Technology skills

Resources for Equity and Engagement

Beyond the chapters of this framework that focus on equity and engagement (Chapters 1 and 2), there are several resources which educators can use as they plan and deliver professional learning experiences to improve equity in mathematics education.

Start callout box

Characteristics of Antiracist Mathematics Educators (from Education Trust West, 2020, Stride 1)

- Design a Culturally Sustaining Math Space
- Center Ethnomathematics
- Make Rigor Accessible Through Strong and Thoughtful Scaffolding
- Prepare Students of Color to Close the Gap in Access to Stem Fields

  Embrace and Encourage Multiple and Varying Ways of Sharing, Showing, and Communicating Knowledge

  Support Students to Reclaim their Mathematical Ancestry

End callout box

- "Improving Education for Multilingual and English Learner Students: Research to Practice" contains a wealth of guidance, resources, and tools for helping teachers and administrators better meet the needs of English learners (https://www.cde.ca.gov/sp/el/er/documents/mleleducation.pdf).

- *A Pathway to Equitable Math Instruction* (https://equitablemath.org/) is an integrated approach to mathematics that centers Black, Latinx, and linguistically diverse students in grades 6–8, addresses barriers to mathematics equity, and aligns instruction to grade-level priority standards. The Pathway offers guidance and resources for immediate use in planning their curriculum, while also offering opportunities for ongoing self-reflection as they seek to develop an anti-racist mathematics practice. The toolkit "strides" (above) serve as multiple on-ramps for educators as they navigate the individual and collective journey from equity to anti-racism. It is a collection of resources to help grades 6–8 Black, LatinX, and linguistically diverse students thrive in mathematics education.

- The California *English Learner Roadmap* (https://www.cde.ca.gov/sp/el/rm/) is an important guide to welcoming, understanding, and supporting English learners in California public schools.

- The Universal Design for Learning (UDL) Guidelines (http://udlguidelines.cast.org/?utm_source=castsite&lutm_medium=web&utm_campaign=none&utm_content=aboutudl) are a tool used in the implementation of the UDL framework, which is a way to improve and optimize teaching and learning for all people based on scientific insights into how humans learn. The three primary principles of UDL are:
    - Provide multiple means of engagement
    - Provide multiple means of representation
    - Provide multiple means of action and expression
- The *Catalyzing Change* series (early childhood and elementary, middle school, high school) from the National Council of Teachers of Mathematics (NCTM, 2018; NCTM, 2020a; NCTM, 2020b) is an important set of texts for professional learning, addressing the following challenges:
    - Broadening the purpose of school mathematics' focus to include the development of positive mathematical identities so that students can make purposeful decisions about their future endeavors
    - Dismantling structural obstacles that stand in the way of mathematics working for each and every student
    - Implementing equitable instructional practices to cultivate students' positive mathematical identities and strong sense of agency
    - Organizing middle school mathematics along a common shared pathway grounded in the use of mathematical practices and processes to coherently develop deep mathematical understanding

Professional Learning Throughout a Teacher's Career

Teachers learn to improve their practice in many contexts: working with students in the classroom, interacting with peers, communications from administrators, attending conferences, enrolling in online courses, and reading publications, to name a few. In this

framework, *professional learning* refers to planned and organized processes that actively engage educators in cycles of continuous improvement guided by the use of data and active inquiry around authentic problems and instructional practices (Coggshall 2012). Darling-Hammond, Hyler, & Gardner (2017, p. v) use the related phrase, *effective professional development*, to mean structured professional learning that results in changes in teacher practices, leading to improvements in student learning outcomes. This section describes important aspects of professional learning at different stages of an educator's career, with particular focus on characteristics of effective professional learning. This is followed by considerations for planning effective professional learning. The section concludes with discussions of various models and strategies for professional learning, with several vignettes illustrating the models and their incorporation of the characteristics of effective professional learning.

Table 9.1, adapted from the National Comprehensive Center for Teacher Quality's publication *Toward the Effective Teaching of New College- and Career-Ready Standards: Making Professional Learning Systemic* (Coggshall, 2012), summarizes key shifts in thinking about professional learning that will help improve teaching practice.

Table 9.1

MOVING FROM	MOVING TOWARD
Believing that professional development is some people's responsibility	Believing that professional learning focused on student learning outcomes is everyone's job
Thinking individual goals for professional development are separate from school site and district goals	Aligning individual goals with school site and district goals to provide greater coherence
Using professional development as a means of addressing deficiencies	Embedding professional learning in continuous improvement
Seldom addressing standards for professional learning	Using standards for professional learning
Providing professional development that	Embedding professional learning in

MOVING FROM	MOVING TOWARD
takes place outside of school, away from students, and is loosely connected to classroom practice	the daily work of teaching so that staff can learn collaboratively and can support one another as they address real problems and instructional practices of their classrooms
Engaging staff in professional development unrelated to data and the continuous improvement process	Engaging staff in a cycle of continuous improvement, guided by the use of active inquiry and multiple sources of evidence
Providing one-shot or short–term professional development with little or no transfer to the classroom	Sustaining continuous professional learning through follow-up, feedback, and reflection to support implementation in the classroom
Limiting professional development based on scarce resources and discrete funding sources	Dedicating and reallocating resources to support professional learning as an essential investment

Source*:* (Coggshall, 2012)

Teacher Preparation

Since CA CCSSM-aligned instruction is different in significant ways from the school mathematics experience of most teachers, the phases of new teacher preparation and induction are key factors in providing a pipeline of teachers with the skills and knowledge to provide high-quality CA CCSSM-aligned instruction. Educators of pre-service teachers need to align their programs to reflect the authentic-context, big-idea based instruction described in this framework so that pre-service teachers have the opportunity to experience it as learners. Factors to consider in the development of CA CCSSM-aligned teacher preparation programs include the following:

- Early field experience hours that are dedicated to observing and interacting with students and teachers in authentic mathematics classroom environments

- Student teaching opportunities that include content-rich experiences and integrated learning experiences
- Mathematics and mathematics methods classes that address mathematics as a collection of tools and lenses for making sense of authentic contexts, with emphasis on learning mathematical ideas through the mathematical practices and active-learning pedagogy rather than passive lecture
- Mathematics and mathematics methods classes that develop mathematics through asset-based, culturally- and linguistically-relevant and sustaining pedagogy
- Mathematics methods classes that address pedagogical content knowledge that facilitates student conceptual understanding of content standards over time and how to address incorrect, developing, and alternative student conceptions of those ideas
- Student teaching experiences with mathematics teachers who are effectively incorporating CA CCSSM
- Effective examples of the development of mathematical ideas through the investigation of authentic contexts and problems (in both pre-service teacher course work and student teaching)
- Mathematics methods classes that address how to organize instruction around big ideas and meaningful investigations, rather than isolated standards
- Mathematics and mathematics methods classes that acknowledge the exclusionary history of the mathematical sciences, explore mathematics from many cultures in ways that do not center European and European-American mathematics as the norm and other cultures' mathematics as "other," and that treat exclusionary practices and narratives as one of mathematics education's biggest challenges
- Mathematics methods classes that make evident ways in which language and content are interconnected and mutually reinforcing: one cannot develop without the other. Language needed for disciplinary thinking and concepts should not be

taught in isolation, but in the context of what students need to know to access and communicate mathematical thinking. Opportunities to practice language and communicate understanding must be integrated (e.g., students have the opportunity to gain ideas from a discussion or a reading before writing)

Additionally, mathematics education faculty and other educators (e.g., university field advisors, master cooperating teachers) who provide pre-service instruction must be grounded in the knowledge and skills within the context of CA CCSSM to facilitate their students' (pre-service teachers) ability to address the vision of the CA CCSSM. Other publications are also important resources for guiding the design of high-quality teacher preparation programs, including the Learning Policy Institute's *Effective Teacher Professional Development* (Darling-Hammond, Hyler, & Gardner, 2017), *Preparing Teachers—Building Evidence for Sound Policy* (NRC, 2010), *Powerful Teacher Education, Lessons from Exemplary Programs* (Darling-Hammond, 2006), and NCTM's Professional Development Guides (NCTM, n.d.).

Induction for New Teachers

Teaching is hard and thoughtful work. It is not uncommon for new teachers to feel isolation and burdened by the demands (both managerial and instructional) of preparing for and working in a classroom. Yet, the implementation of effective preparation and support programs specifically tailored to the needs of new teachers can alleviate these issues to a large degree. The following considerations can provide support for prospective teachers of mathematics:

- Redefine the professional dynamics of the teacher induction process by pairing beginning mathematics teachers with experienced mathematics teachers who can act as mentors rather than delegators. This connection may help address the need for inclusion and community, and may provide the new teacher a sense of ownership of the content and a sense of belonging in the mathematics department, leading to greater teacher retention.

- Recognize and support the need for elementary teachers to receive math-specific support and mentoring.
- Ensure that beginning mathematics teachers have comparable access to mathematics teaching resources (including technology, teaching spaces, and materials for hands-on instruction) as other mathematics teachers in the school.
- Involve new teachers in available Professional Learning Communities, Lesson Study, or the like, particularly math-specific ones, in order to promote and aid regular reflection on their practice (Fulton, Britton, & Doerr, 2010).
- Encourage new teachers to attend mathematics teacher conferences, institutes, and workshops (and financially support them to do so).
- Ensure that beginning teachers understand who their students are, in particular their emerging multicultural learners, their interests, aspirations, and backgrounds and how to use those as resources for learning.

Ongoing Professional Learning for In-service Teachers

Characteristics of Effective Professional Development

Though there are many approaches to professional development—along with multiple aspects to each approach—some strategies and components have been shown to be more effective than others. NCTM in *Principles to Action* (2014) connects education research to teaching practice with professional learning materials to help educators learn specific, research-based teaching practices. The Learning Policy Institute's review of 35 rigorous studies on the implementation of professional development for teachers noted several elements of effective professional development (Darling-Hammond, Hyler, & Gardner, 2017). These elements, described below, include a focus on the following:

- content
- active learning
- collaboration
- modeling

- coaching
- feedback and reflection
- sustained engagement

Content Focused

Professional development in any discipline has been found to be most effective when the content knowledge in that area—in this case mathematics—is a primary focus. Teachers must have opportunities to explore mathematical big ideas through rich, authentic, culturally-relevant tasks in order to both deepen their own understanding of mathematics and better anticipate the challenges students might encounter and the strategies they may rely on to respond to them. These big ideas include the mathematical practices as central aspects of mathematics, equal in import to content standards. Professional development that introduces perspectives or teaching approaches without intentional connections to mathematics is unlikely to bring about much change in teachers' practice. Professional development that blends pedagogical and learning knowledge with mathematics knowledge has much more potential to result in powerful changes in students' learning experiences than that which focuses on pedagogy or content knowledge separately.

Many teachers have experienced mathematics as a set of procedures to be memorized. This narrow understanding makes access to opportunities to experience mathematics differently themselves all the more important, lest their own students have their mathematics identities shaped by similarly limited experiences of mathematics. As described in Chapter 1, the goal is that students achieve conceptual understanding, problem solving capacity and procedural fluency (in the full sense of the word fluency introduced in Chapter 1) in mathematics. When teachers work on rich, authentic, culturally-relevant mathematics tasks—through which they can ask their own questions, reason and communicate with others, develop curiosity and wonder—they start to see mathematical connections that they may never have seen before. This often changes

teachers' relationships with mathematics, which is an important precursor to changing their teaching (see also Anderson, Boaler & Dieckmann, 2018). This experience takes time and needs to be carefully organized, with teachers working together on mathematics in a supportive environment with an expert facilitator. Face-to-face professional development is the ideal way to encourage this experience, but online courses can also provide this experience, especially when teachers receive funded time to take the courses in groups.

Based in Active Learning

Teachers benefit most from professional development that engages them in the process of actively designing and trying teaching strategies, and provides them with opportunities to engage in the same style of learning they are designing for their students. Such professional practice relies on authentic artifacts, interactive activities, and other strategies to provide deeply embedded, highly contextualized professional learning. This approach moves away from traditional learning models and environments that are lecture based and fail to connect to teachers' classrooms and students. Instead, teachers should have opportunities to make sense of student thinking (in order to assess students' funds of knowledge and other assets—such as reasoning and communication practices—that will help drive teacher actions), reflect on their own and one another's instructional practices, and discuss connections to their own classroom. Classroom video is a powerful resource for such reflections and discussions. For example, professional development may include opportunities to watch videos showing linguistically and culturally diverse communities of English learners working to high levels with an expert teacher. Videos and other records of practice such as student work, should be at the center of professional development opportunities.

Includes Collaboration

Effective professional development requires time and resources for teachers to share ideas and collaborate in their learning, often at the school level. Working collaboratively

allows teachers to create professional learning communities that can positively change the culture and instruction at a classroom, grade, department, school, or district level. As teachers work together on mathematics instruction, they experience the collaborative, connected mathematics experience as a template for their own classrooms. They can also share experiences, including challenges, successes, and insights, to support one another in planning and implementing lessons. Professional learning communities are also important places to consider ways in which mathematics instruction can recognize students' cultural and linguistic assets, to make contexts and problems ever-more authentic for students.

Uses Instructional Examples

Seeing lessons, tasks, and curriculum in action is a powerful tool for providing teachers with opportunities to see best practices first hand. Teachers may view examples that include lesson plans, unit plans, sample student work, observations of peer teachers, and video or written cases of teaching, such as the many vignettes and snapshots presented in this framework. Teachers benefit from opportunities to discuss examples of teaching, reflect on current practices, and make connections to their own classrooms.

Effective professional learning must build teachers' capacities to notice, analyze, and respond to students' thinking (NCTM, 2014, p. 101), and professional learning built around artifacts of practice such as student work (written, video, or other) provides time and support to develop these capacities.

Provides Coaching and Expert Support

Implementing new teaching approaches can shift particular classrooms, schools, or even districts. Fortunately, coaching and expert support—especially from district and county mathematics coaches—has proven extremely effective to respond to these changes when it is structured around a particular purpose (for example, adopting new curriculum or implementing specific new instructional practices) and is aligned with

school-wide goals and priorities. Well-trained peers and teacher leaders with expertise in particular approaches can be powerful facilitators of growth in encouraging, modeling, and sharing insight—particularly when supported by administration and appropriate structure. These leaders can spend time observing teachers' instructional practices, recognize assets that teachers can build on, and work with teachers toward ever-growing capacity to implement rich, student-centered mathematics lessons.

Includes Feedback and Reflection

High-quality professional development ensures teachers are afforded dedicated time to think about, receive input on, and make changes to their practice. They can facilitate reflection and solicit feedback, both of which enable teachers to establish and refine realistic goals of changing practice as they move toward expert visions of practice.

Has a Sustained Duration

Effective professional development provides teachers with adequate time to learn, practice, implement, and reflect upon new strategies that facilitate growth in their practice. Professional development which engages teachers in making incremental changes over time (and reinforces existing practices) can bring about lasting positive changes.

Planning for Effective Professional Learning

Achieving this framework's vision of mathematics education will require improved systems of professional learning. Teachers, specialists, paraprofessionals, and school and district leaders should identify personal and collaborative learning goals that articulate across grade levels and departments, focusing on curriculum, instruction, and assessment strategies that embrace the vision of the CA CCSSM and this framework. The schools, districts, and other local educational agencies (LEAs) must become "learning organizations" (Senge, 1990) that are engaged in continuous improvement around the teaching and learning of mathematics. At every level (grade, department,

school, district) educators must share a vision that focuses on student learning, collaboration, collective inquiry, shared practices, reflection, and results (Louis, Kruse, and Marks, 1996; DuFour, 2004; Hord & Sommers, 2008).

County offices of education, districts, schools, and other LEAs providing professional learning can use the report Effective Teacher Professional Development (Darling-Hammond, Hyler, & Gardner, 2017) as a resource for planning these types of learning experiences. This report gives much more detail about the features of effective professional learning described above.

Another resource for those designing professional learning opportunities is *Professional Development Design Framework* (Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N., & Hewson, P. W., 2010). Through their research with national professional developers, Loucks-Horsley and her colleagues found that effective programs had several common characteristics. They were designed to meet various factors, to change over time, and to adapt to particular goals and contexts. They did not rely on formulas; instead, the designers used a process of thoughtful, conscious decision making. The authors used these factors and processes to create the framework as seen in figure 9.X below.

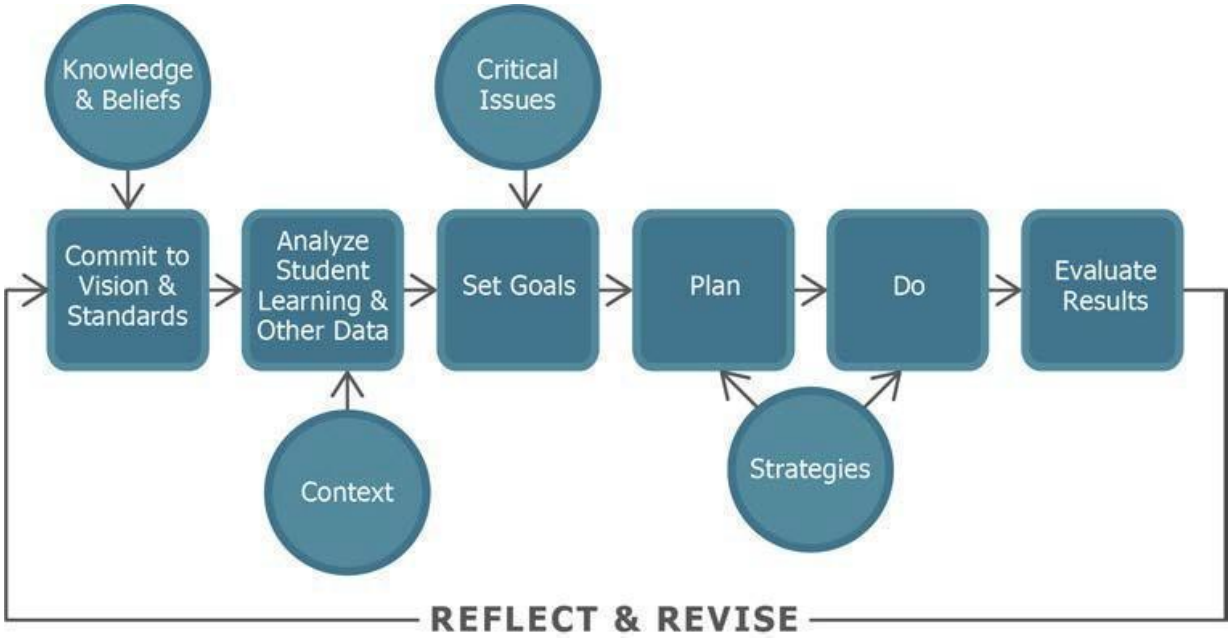


Figure 9.X. Professional Development Design Framework

Source: Loucks-Horsley et al., 2010.

At the center of the design framework, illustrated in the six squares connected with horizontal arrows, is a planning sequence that includes the following topics: (1) committing to a vision and a set of standards; (2) analyzing student learning and other data; (3) goal setting; (4) planning; (5) doing; and (6) evaluating. The circles above and below the planning sequence represent important inputs into the design process that can help designers of professional learning make informed decisions. These inputs prompt designers to consider the extensive knowledge bases that can inform their work (knowledge and beliefs), to understand the unique features of their context, to draw on a wide repertoire of professional development strategies, and to wrestle with critical issues that instructional reformers will encounter.

While there is no exact starting place for using the design illustrated in Figure 9.X, effective planning should avoid starting with strategies—though they may seem most appealing. Instead, the use of evidence (derived through questions such as, What are the assets? or, What are the needs?) is encouraged. Additional considerations should

be made, such as thinking about short- and long-term approaches (up to five years), considering teacher career trajectories, and supporting teachers accordingly (Task Force on Educator Excellence, 2012).

However, those developing professional learning must also remain mindful of the need to stay flexible and adaptive, and they should include openness to refining their ideas as they evaluate the implementation process. As the design and implementation phases are taking place, recommendations from Innovate: A Blueprint for Science, Technology, Engineering, and Mathematics in California Public Education (STEM Task Force, 2014) and the characteristics of effective professional learning should also be considered during the design phase.

For consideration: while the Professional Development Design Framework in Figure 9.X is arranged as a linear and sequential model, it need not be employed as such. What is most important is to pay attention to the four core design inputs, where they impact the design of the program, and how they are addressed during implementation.

Models and Strategies: Effective Professional Learning

The characteristics of effective professional learning can be implemented through many professional development models and strategies, including the following:

Models

- Professional Learning Communities (PLCs): opportunities for teachers to collaborate with each other, and for administrators to collaborate with their teachers, in a team setting
- Communities of Practice are “...groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly” (Wenger-Trayner & Wenger-Trayner, 2015). In educational settings, PLCs are often site-based, and Communities of Practice often connect educators

across sites, helping provide additional contacts and resources for improving practice.

- Classroom coaching: A **mathematics coach** is an individual who is well-versed in mathematics content and pedagogy and who works directly with classroom teachers to improve student learning of mathematics (Hull, Balka, & Miles, 2009).
- Lesson Study (see below)
- Mathematics Labs: Collaborative design and instruction cycle, similar to Lesson Study but with collaborative instructional decisions even during the lesson's implementation (Kazemi, Gibbons, Lewis, Fox, Hintz, Kelley-Petersen, Cunard, Lomax, Lenges, & Balf, 2018)
- Content-intensive institutes with follow-up workshops (see below)

Strategies

- **Backwards design**: importance of student learning outcomes in lesson design
- Implementation of and alignment with the guidelines of Universal Design for Learning (UDL)
- Stride 5: Sustaining Equitable Practice of *A Pathway to Equitable Math Instruction* (Education Trust West, 2020) provides extensive guidance for equity-focused classroom coaching
- Networking and community building around mathematics instruction: Mathematics Teacher Circles (https://www.mathteacherscircle.org/), teacher fellowship programs (e.g., https://knowlesteachers.org/teaching-fellowship/teaching-fellows-program), mathematics professional associations (e.g., California Mathematics Council, https://www.cmc-math.org/)
- Partnerships with university mathematics and mathematics education faculty: Bridging the research–practice divide

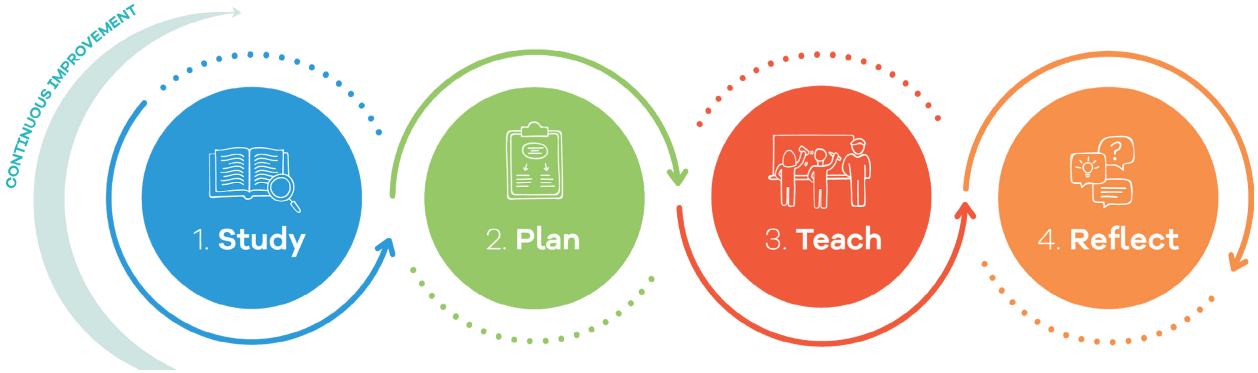
Three models that are supported by research into effective professional development in mathematics are explored below. The first, Lesson Study, offers sustained content-focused courses with school-year follow-up, and coaching. In a survey of the effectiveness of 643 professional development models, only two models were found to have a significant positive effect on students' learning—lesson study and sustained content-focused summer courses with pedagogy-oriented structured academic year follow-up (Gersten, Taylor, Keys, Rolfhus, & Newman-Gonchar, 2014). Coaching models are very common in California schools, but "...there is little empirical evidence that coaching improves teacher practice" (Desimone & Pak, 2017). However, some structured coaching models show more promise for instructional improvement than individual one-on-one models (Gibbons, 2017).

Lesson Study

Lesson study is a type of professional learning where teachers engage in an inquiry cycle that supports their ability to experiment, observe and improve their teaching by collaboratively researching, creating, teaching/observing, and revising a lesson. Lesson study, which originated in Japan, has been shown to be an effective model for professional development with its deliberate focus on planning and teaching practice as well as inquiry, creativity, and collaboration (Lewis & Hurd, 2011).

The proven effectiveness on student learning led the California Mathematics Project (CMP) to formally adopt lesson study as a preferred means of professional development in 2018, and eventually spearhead the creation of the California Action Network for Mathematics Excellence and Equity (CANMEE, https://cmpso.org/canmee/). CANMEE supports California schools and districts in their implementation of high-quality lesson study. The Lesson Study Group at Mills College (https://lessonresearch.net/) provides many online resources to support such implementation.

The lesson study cycle consists of four phases (adapted from https://lessonresearch.net/about-lesson-study/what-is-lesson-study/).



In the Study phase, a team of teachers collaborates to:

- Identify long-term goals for students
- Choose the subject and unit to investigate
- Study standards, research, and curricula

In the Plan phase, using insights from the Study phase, the team:

- Examines the unit and chooses one lesson to plan in depth
- Articulates the lesson goals
- Tries the lesson task and anticipates student thinking
- Identifies data to be collected during the lesson

In the Teach phase, the team puts that lesson into action:

- One team member teaches the lesson
- Other team members observe and record student thinking and learning

In the Reflect phase, the team then reflects on their work by:

- Meeting after the lesson to discuss data on student thinking and learning

- Having an outside specialist provide further commentary
- Reflecting on what they learned during the cycle as a whole

Some or all of these phases are often repeated by a team as a team often wishes to redesign a lesson based on realizations made in the Reflect phase, and teach it again to another class of students.

It is important to note that the “product” of a lesson study cycle is more than a refined lesson plan: Team members deepen their understanding of content and student thinking, their commitment to collaboration, and their ability and inclination to base instructional decisions on evidence of their students’ thinking.

Lesson Study Vignette

Grade level: Second

Equity focus: Linguistically and culturally diverse English learners’ productive language use in mathematics

Source: The California Action Network for Mathematics Excellence and Equity (CANMEE) Steering Committee, adapted

The second-grade teachers at 54th Street Elementary met during their Professional Learning Community time to discuss the performance of their emerging multicultural learners in mathematics. Each teacher noticed that their English learners were having difficulty explaining their solutions to mathematics problems orally and in writing. They invited the English language development (ELD) specialist to the meeting to hear their concerns and obtain suggestions for addressing the students’ needs.

The ELD specialist had recently observed a lesson at another elementary school focused on equity. The ELD specialist suggested that the second-grade teachers consider participating in a lesson study focused on building the agency of their

multilingual students. The teachers decided to engage in a lesson study cycle of 30 hours and followed the lesson study model of study, plan, do/test, and reflect.

As part of the equity focus of the CANMEE lesson study process, each teacher selected four designated English learners as focal students from their classes and interviewed them to determine their strengths and challenges in mathematics. Based on the content of interviews and classroom observations, the teachers drafted assets-based descriptions for each, then met and shared their focal student descriptions.

During the Study phase of their lesson study, the teachers read literature that centered on effective practices for English learners, such as the *English Language Arts/English Language Development Framework* (CDE, 2014), the *English Learner Roadmap* (CDE, 2017), *Pathway to Equitable Math Instruction* (Education Trust West, 2020), and important research (Moschkovich, 2012; Ramirez & Celedón-Pattichis 2012). As part of the Plan phase, teachers designed a mathematics lesson with a task that required students to record their thinking in a journal, and share their ideas with a partner. One of the goals for the focal students was to increase their productive language skills. The teachers engaged in the mathematics task themselves to anticipate both productive and unproductive student strategies. The teachers developed questions to ask those students who used unproductive strategies, and consulted with the ELD specialist for additional resources. The specialist posed questions to allow the teachers to do the thinking.

In the do/test phase, one of the teachers on the team volunteered to teach the lesson while the other teachers observed the lesson to determine the effect of the lesson they designed. An outside expert in mathematics content was invited to provide feedback on the mathematics content of the lesson, serving as the mathematics commentator. The ELD specialist served in the role of equity commentator. The ELD specialist observed the focal students' interaction with the lesson and peers, and their productive language skills—in particular, aspects of the lesson design that seemed to facilitate productive

language opportunities. The second-grade teachers also invited other stakeholders, including colleagues at the school and parents, to observe the public lesson.

After the lesson was taught, as part of the Reflect stage, the team of teachers shared their thoughts and observations about the implementation of the lesson, and identified ways to improve practice moving forward. The mathematics and equity commentators shared their observations of the lesson and provided suggestions for next steps. Other observers also made comments about the lesson.

At the end of the cycle, the second-grade teachers reflected on the professional learning experience. They noted the value in the ability to collaborate with their peers about a problem of practice that was specific to their school. The teachers also felt that the support from the ELD specialist was critical to their success. They all noticed an increase in agency among the focal students as a result of the lesson study process. Lastly, the second-grade teachers noted feeling more confident about their ability to meet the needs of their students who are emerging multicultural learners.

Content-focused workshops with follow up

"One and done" professional development sessions have shown little impact on teaching practice or student learning (Darling-Hammond, Hyler, & Gardner, 2017). In addition to lesson study, sustained content-focused professional courses/workshops with school-year pedagogy-focused follow up have also demonstrated positive impact on student learning (Gersten et al., 2014). There are several partner organizations in California that work with districts and schools to provide these opportunities.

The California Mathematics Project (CMP) is a state-supported K–16 network dedicated to providing students a rich, rigorous, and coherent mathematics curriculum taught by competent and confident mathematics teachers who foster all students' proficiency in mathematics—achieving equity in quality (https://cmpso.org/). The 19 regional sites of the CMP are co-led by university faculty and teacher leaders, creating high-quality

professional learning focused on pedagogical and content knowledge development. CMP enhances teachers' mathematical content knowledge and pedagogical content knowledge that is aligned to the CA CCSSM and the 2013 *Mathematics Framework*. All teachers and students become competent mathematical thinkers as they investigate, conjecture, and justify.

Youcubed (https://www.youcubed.org/) is a nonprofit based at Stanford University with the mission directed at "Inspiring math success for all students through growth mindsets and innovative teaching." Partner districts work with Youcubed to design sustained growth-mindset based professional learning opportunities. Youcubed also provides extensive online resources.

The Silicon Valley Mathematics Initiative (SVMI; https://svmimac.org/) is a comprehensive effort to improve mathematics instruction and student learning. The Initiative is based on high performance expectations, ongoing professional development, examining student work, and improved mathematics instruction. The Initiative includes a formative and summative performance assessment system, pedagogical content coaching, and leadership training and networks. Its professional development offerings and other resources are available to member districts and schools throughout California.

Professional Learning Vignette: California Mathematics Project

[California Math Project vignette under development]

Professional Learning Vignette: Tulare County–Youcubed partnership

This vignette describes a model of professional learning which combines a focus on mathematical mindset and content knowledge, through a model of paid time where teachers can learn and plan together with shared goals and resources.

The Tulare County Office of Education partnered with Youcubed in offering a blended model of professional learning for teachers and leaders across 11 school districts. The partnership was called the Central Valley Networked Improvement Community (CVNIC). County leaders chose fifth grade as the focus of the work, as very low percentages of students in fifth grade either met or exceeded proficiency on the California Assessment of Student Performance and Progress (CAASPP). Table x shows the percentages of students involved in the initiative by ethnicity, socio-economic status, language learning and their proficiency results on the fifth-grade CAASPP tests:

Regional Schools	Student N	Percent Latino	Percent English Learner	Percent Low SES	Percent Proficient (All Students) on CA Grade 5, Test* (2016)
School 1	572	72%	28%	83%	8%
School 2	410	68%	35%	86%	17%
School 3	712	98%	64%	97%	7%
School 4	624	95%	63%	96%	8%
School 5	445	28%	42%	21%	5%
School 6	487	19%	68%	19%	3%
School 7	687	11%	58%	11%	4%

During the year-long partnership, teachers and their administrators were provided paid time to complete a Youcubed course online. Upon completion of the course, called “How to Learn Math” (https://www.youcubed.org/how-to-learn-math-for-teachers/), teachers met in groups to discuss learning and plan classroom changes. The meeting time was facilitated by county office leaders who led full-day sessions centered on mathematics collaboration. The network focused on implementing structures that reinforced the importance of growth mindsets in mathematics and ways for students to see mathematics as a connected, visual subject, with classroom strategies that fostered this approach.

Many teachers shared that particular students, especially those designated as multilingual learners, had developed the idea they did not “have a math brain” and that

mathematics was a set of procedures to memorize. This factored in their achievement levels.

The professional development sessions conducted by the county included engaging the teachers with rich mathematics tasks that were visual and showed the connected nature of mathematics (e.g., https://www.youcubed.org/tasks/). The teachers' work was informed by the research promoting the importance of struggle for brain development, and they were reminded that students remained capable of learning anything. The teachers in the networked community agreed to begin the school year with the "week of inspirational math" lessons (www.youcubed.org/week-inspirational-math/). The schools conducted diagnostic surveys to learn about the students and their ideas about mathematics. These surveys were conducted at the beginning of the school year, and repeated again at the end of the year of the intervention.

Each time the teachers took a lesson from the online course, they met to discuss the changes the lesson would inform in their classrooms. As the year progressed the teachers continued to include the use of rich, visual, creative mathematics tasks with increasing frequency; this altered their textbook tasks, inspiring more flexible uses, and allowing them to rely more on students' ideas, how they use them, and share how they strategize in mathematics. The teachers reflected that this gave students—and the teachers, too—new access to understanding. As one teacher shared:

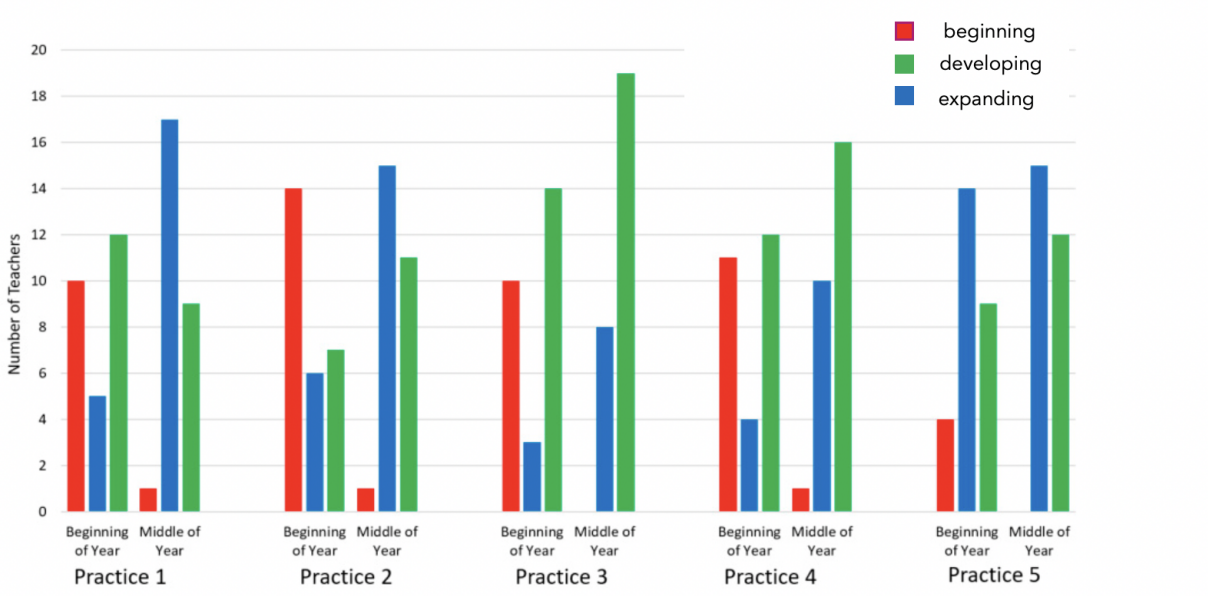
> When I first started this journey, I was always doing the algorithm because that was my safety net. Now I'm thinking, "Okay, how am I going to draw this? How do I visually see this?" Now I understand why the algorithm works, because I now have this totally clear picture in my head. Which has been a really good thing when it comes to things like fractions. And for these kids, it's like, "Oh, that's why it works."

Another teacher noted:

> Oh, the visuals.... They love that too, cause with their ideas of how it would form, and how they would build. I do it periodically. Just throw up a visual with different things and say, “Okay, what do you see? What don’t you see? What might you see? What could be the next thing?”

The county leaders and the Youcubed team developed a mindset guide to help teachers and leaders understand the important aspects of a mathematical mindset. The guide includes advice for giving mindset messages, using rich tasks and emphasizing mathematical and student connections, and assessing students in ways that are compatible with a mindset approach. County officials observed classrooms at different intervals and recorded the teachers’ practices in relation to the features of the guide. Table 9X shows that the teachers developed their practice in relation to all five features of the guide. At the beginning of the year, high proportions of teachers were at the “beginning” level of the five classroom features, by the middle of the year, more teachers were at the “developing” and “expanding” levels, with three of the features reaching significance levels. For example, the teachers’ practice on the “nature of mathematics” significantly improved ($t = 3.03$, $p = 0.005$).

Table 9X



The Mindset Guide, used for training teachers and as an observational tool:

Mathematical Mindset Practice 1: Growth Mindset Culture

Mindset	Beginning	Developing	Expanding
Mindset Messages	Brain and belief messages are never given or only to some students	Belief messages are given occasionally or too generically	Brain and belief messages are given in a meaningful way: “I know you can do this,” “As you learn this pathway forms in your brain”
Praising the Learning Process	Praise is focused on answers rather than effort and progress in thinking	Praise is sometimes focused on effort and process	Effort, ideas, and strategies are consistently recognized and praised
Students’ Mindsets	Students talk about some people being “math people” and some not	Students convey a mix of confidence and doubt in themselves	Students show self-belief and confidence

Mathematical Mindset Practice 2: Nature of Mathematics

Mindset	Beginning	Developing	Expanding

Open Tasks	Tasks are relatively closed, emphasizing procedures with little reasoning	Rich tasks are occasionally used	Tasks are mathematically rich in reasoning opportunities, allowing for different approaches and visuals
Reasoning and Multiple Perspectives	Maths work does not include reasoning, visuals, or multiple perspectives	Occasionally multiple methods and visuals are elicited and explored	Students use and share different ideas, visuals, and methods and use ownership words (e.g., “my method”)
Depth Over Speed	Strong emphasis on speed, memorization, and correct answers	Occasional emphasis on speed, memorization, and correct answers	Emphasis is on depth, creativity, visuals, and mathematical beauty

Mathematical Mindset Practice 3: Challenge and Struggle

Mindset	Beginning	Developing	Expanding
Mistakes	Complete and correct work is emphasized, mistakes are discouraged	Mistakes are acceptable but not explored	Mistakes are valued, students are comfortable sharing even if unsure
Struggle & Persistence	Students expect and rely on teacher assistance when they struggle	At times struggle is celebrated, at others students are led to a solution	Struggle is valued; e.g., “this is the best time for brain growth.” Students persist longer
Questioning	Questions are low-challenge or narrowly focused	Deep-thinking questions are occasionally used	Questions are open and encourage multiple methods, ways of seeing, and thinking

Mathematical Mindset Practice 4: Connections and Collaborations

Mindset	Beginning	Developing	Expanding
Mathematical Connections	Maths is presented as a set of disconnected ideas	Connections are implied but seldom discussed	Connections between ideas, methods, and representations are highlighted and explored through visuals, movement, and creativity
Connecting in Small Groups	Student discussion is not encouraged	Student discussion is encouraged but only some students take part	Students collaborated and build off each other’s ideas and all students are involved

Connecting as a Whole Class	No opportunities for whole-class discussion	Class discussion is encouraged, e.g., "Does anyone want to respond to [blank]'s idea?" but most interactions are teacher-student.	Students talk directly to each other; the teacher is just one member of the mathematical community

Mathematical Mindset Practice 5: Assessment

Mindset	Beginning	Developing	Expanding
Nature of Feedback	Harsh grading on a curve, ranking, no revisions, punitive	Standards-based grading – with feedback on standards met. Revisions are not allowed	Assessment is used formatively, e.g., verbal, written, and ongoing feedback on specific learning goals. Revisions are encouraged
Frequency of Testing and Grading	Grades and tests/quizzes are frequent; performance culture	Grades and other summative measures are only given at the end of the unit	Learning culture with diagnostic feedback
Multiple Forms of Assessment	Assessment is based on tests, quizzes, and homework. Focus is on answers only	Assessment includes more multidimensional evidence of learning, not only answers	Formative assessment valuing a broad form of mathematics – e.g., example visuals, making sense, multiple methods

(These graphics and the videos embedded in them are available online at https://www.youcubed.org/mathematical-mindset-teaching-guide-teaching-video-and-additional-resources/.)

The blended model of professional learning led to several changes over the course of the school year. Importantly, the teachers who took part in the network changed their own views of themselves—prior to taking the online course many teachers believed they could not be good at mathematics, and that mathematics was a set of procedures. As teachers changed these ideas about themselves, and about mathematics, they were able to teach differently. One of the teachers reflected on this personal change saying,

> "I thought it was going to be great for the kids, I never expected it to change me, that's been my greatest revelation in all of it."

By the end of the school year the students of the teachers in the network achieved at significantly higher levels on the mathematics portion of the CAASPP. The focus on mindset particularly raised the achievement of girls, language learners, and economically disadvantaged students (see Anderson et al, 2019). A survey taken by over 400 students showed that students significantly changed their beliefs, particularly changing their view that only fast thinkers could be successful, and their belief that only some people could be successful ($t = -8.69$, $p < 0.001$).

Teachers reflected that changed classroom environments—those that valued struggle and multi-dimensional mathematics—deeply and positively impacted their students:

> "The kids were thrilled, going 'Oh my gosh, he's doing it like that? It's OK that we struggle? It's OK we think differently?'"

> "I just want you to know this has meant a lot. Seeing how positive the kids are about their learning now has made a world of difference. The confidence they have is unlike anything I have ever seen."

Notably, the teachers also shared that the change in their teaching had started with a change in their own relationship with mathematics.

Analyses of the impact of the blended professional learning highlighted the importance of the combined attention to mindset (valuing brain growth and struggle), and to mathematics—working with teachers to open-up tasks and value multidimensional work (visual, numerical, verbal, modelling). The time that teachers were given to work together, access online and face-to-face professional development, and experience creative mathematics themselves, was critical to the success of the network. The blended approach and the details of teacher and student change is explained fully in Anderson et al, 2019, https://www.mdpi.com/2227-7102/8/3/98.

Structured Coaching

Instructional coaching best contributes to school-wide mathematics instructional improvement when it is used as a tool to support the *collective* learning of teachers (Gibbons, 2017). In other words, the characteristic of effective professional learning that "provides coaching and expert support" does not stand alone; designating a "good mathematics teacher" as a coach has not proven to improve teaching practice by itself. Coaching is effective when it is structured to provide more than a model/co-teach/you teach feedback loop: "Coaches need to engage teachers in fundamental dialogue about mathematical content, mathematical learning, and student understanding" (Campbell & Griffin, 2017). Thus, coaching is effective when it is part of a broader professional learning plan that incorporates most or all of the other characteristics of effective professional learning, as in the following vignette.

[Coaching resources and vignette under development]

Teacher Leadership

Ultimately, successful development and implementation of effective professional learning for teachers relies on expertise, which requires district capacity. However, the use of outside expertise can, over time, diminish the district's capacity to build internal leadership. Conversely, using in-house personnel that may lack the necessary expertise is not effective for creating lasting, meaningful changes that students are entitled to received. Districts must consider ways to build teacher, curricular, and administrative leadership, with the assistance of outside sources, to strengthen their long-term capacity to improve mathematics learning. Every district will have some teachers who show more interest in and more action around seeking opportunities to develop personal capacity to provide authentic mathematics learning opportunities. Identifying these "early adopters" and supporting their learning—as well as leadership roles in supporting other teachers—can be an effective way to strengthen a school or district's professional learning networks for mathematics.

This section begins with the development of teacher leadership as a core strategy for supporting improvement in teaching and learning, because research indicates that leadership and support are required in order for professional learning experiences to be turned into changes in teaching and learning practices (Lieberman & Miller, 2008; Weiss & Pasley, 2009). Teacher leadership is associated with increased teacher learning and creating collaborative professional cultures (York-Barr & Duke, 2004; Werner & Campbell, 2017), as well as being positively related to increased student achievement (Waters, Marzano, & McNulty, 2003).

Teacher leadership addressed in this section resonates with a definition of leadership from Julian Weissglass (1998), which states: “Teacher leadership is about taking responsibility for what matters to you.” In other words, teacher leaders include every teacher—those who are seeking or are designated teacher leaders, department chairs, teachers on special assignment, mentors and coaches, etc. Everyone has the capacity for leadership, and one goal of mathematics teacher leadership is to have many, rather than a few, people leading creatively every day and in all aspects of their lives (Kaser, Mundry, Stiles, & Loucks-Horsley, 2013). This view of teacher leadership differs from the traditional view in that leadership is not about power and authority. Instead, it embraces five practices of exemplary leaders (Kouzes & Posner, 2003), as listed in Table 9.X.

Table 9.X. Practices of Exemplary Leadership

PRACTICES OF EXEMPLARY LEADERS	DESCRIPTOR
Challenging the process	Searching for opportunities to change the status quo and innovative ways to improve
Inspiring a shared vision	Seeing the future and helping others create an ideal image of what the organization can become
Enabling others to act	Fostering collaboration and actively involving others

PRACTICES OF EXEMPLARY LEADERS	DESCRIPTOR
Modeling the way	Creating standards of excellence and leading by example
Encouraging the heart	Recognizing the many contributions that individuals make, sharing in the reward of their efforts, and celebrating accomplishments

Leadership development requires explicit attention, clear expectations, and resources, time, and expertise (Hopkins, Spillane, Jakopovic, & Heaton, 2013; Yow & Lotter, 2016). Mathematics teacher leaders need to continually build their: (1) in-depth understanding of the mathematics content and practices of the CA CCSSM; (2) thorough knowledge of the best practices in teaching and learning based in authentic contexts and problems; (3) understanding of school culture, organization, and politics; (4) understanding of change theory; (5) knowledge of how adults learn; and (6) practices that embrace continuous improvement. Additionally, leaders need skills in facilitation and communication, using data and decision making, and organization, to name a few.

Teacher leaders can take on a variety of roles to help colleagues and other educators, as well as parents, guardians, and community members become more aware of and aligned with improvements in mathematics teaching and learning.

These roles include leading in the areas of (1) instruction and assessment; (2) curriculum and instructional materials; (3) school culture that is supportive and proactive for the implementation of the CA CCSSM (4) community support and advocacy for active, authentic mathematics instruction; and (5) mathematics classroom implementation of the California ELA/ELD Standards. An explicit current in all of these roles must be access and equity for all students.

To develop these knowledge and skill sets, teacher leaders need professional learning targeted toward leadership. Learning experiences are most productive when they occur

over time, provide feedback, are anchored in the practice of instructional leadership, and ground the leaders in mathematics practices and content (Fullan, 2015; Kaser et al., 2013; Darling-Hammond, Hyler, & Gardner, 2017). Districts need to develop leadership programs that embrace these attributes, and/or encourage their teacher leaders to participate in these types of leadership experiences through programs such as the CMP, the Silicon Valley Mathematics Initiative, Youcubed, and the California Mathematics Council.

Teacher leadership can be manifested in many forms, including presenting (at the school site, district, or professional organization level), consulting (as informal specialists for other mathematics teachers), facilitating (site-level department collaboration, Lesson study groups, and district-level efforts such as assessment and vertical alignment choices), and coaching.

The extensive literature on teacher leadership cited in this section provides additional sources for further learning by those seeking to empower and support teacher leaders.

Administrative Leadership for Professional Learning

Administrators play a key role in helping create and sustain a multi-layered system of support for teachers in their pedagogy and professional learning. There are several dimensions to the types of specific support administrators can provide, including having well-informed conversations about teaching and assessment, as well as feedback on instruction and critical conversations about instruction.

Together with their teaching staff and paraeducators, administrators may need to seek opportunities to understand more about the nature of mathematics learning and teaching presented in this framework. In establishing and maintaining regular communication with teachers about their teaching, their students, and the curriculum, administrators play a pivotal role in the confidence and vision necessary to help teachers explore new ways of ensuring all students can engage with mathematics. The

guidance presented in this framework can serve as a starting point in helping to structure these conversations.

Administrators should be aware of this framework's responses to the challenge posed by the principle of coherence. They are: **progressions** of learning across grades (thus, grade-band chapters rather than individual grade chapters), **big ideas**, and **relevance** to students' lives. In particular, the learning progressions chapters (Chapter 3, 4, and 5) highlight the value in building powerful ideas about numbers and data that, over time, grow in meaning and resonate in subsequent grades' topics; and on focusing learning upon productive habits of mind such as exploration, discovery and communication involving mathematics.

Administrators should be aware of the general principles guiding the development of the grade-band chapters (Chapters 6, 7, and 8). In general, these principles include: designing lessons from a small number of big ideas in each grade band; a preponderance of student time spent on authentic problems that engage multiple content and practice standards situated within one or more big ideas; a focus on connections, both between students' lives and mathematical ideas; and strategies between different mathematical ideas of various topics across grade level.

Working with their teaching staff, administrators may need to identify opportunities to learn more about inclusive teaching strategies. Chapter 2 sets out the important qualities of mathematics classrooms that encourage student engagement and equitable outcomes. Through professional workshops, conferences, or other professional learning, administrators can support their teachers in this important learning. They should also draw upon teacher leaders at their school site or within their district who can provide support and knowledge of inclusive teaching approaches, especially those that focus on students who are culturally and linguistically diverse learners and students with learning differences. There are many resources for administrators to learn more about

mathematics teaching and learning to address issues of equity and promote social justice, including the following:

- TODOS: Mathematics for ALL (https://www.todos-math.org/professional-dev)
- A Pathway to Equitable Math Instruction (https://equitablemath.org)
- Just Equations (https://justequations.org/resource/branching-out-designing-high-school-math-pathways-for-equity/)
- Math Success for All (https://sites.google.com/placercoe.k12.ca.us/mathandswd/math-success-for-all)
- Youcubed.org (youcubed.org)
- Batmath (batmath.org/)
- The Quality Professional Learning Standards, developed by the California Department of Education (https://www.cde.ca.gov/pd/ps/qpls.asp)
- The Teaching for Robust Understanding (TRU) Math Framework (https://truframework.org/)
- The Strategic Education Research Partnership (https://www.serpinstitute.org/)
- Inside Mathematics (https://www.insidemathematics.org/)
- Mathematics Assessment Project (https://www.map.mathshell.org/index.php)
- Development and Research in Early Math Education (DREME) https://dreme.stanford.edu/

An important idea conveyed in this framework is that all students deserve access to high-level mathematics curriculum. The tradition of ability grouping and tracking students in the elementary and middle years has resulted in widespread racial inequalities and the filtering of many students out of pathways that are perceived as mathematics-dependent (including science, technology, engineering, and mathematics). In addition, there is research demonstrating negative effects of ability grouping and tracking on those in the “high” group or track as well (Becker, Neumann, Tetzner, Böse, Knoppick, Maaz, Baumert, & Lehmann, 2014; Mulkey, Catsambis, Steelman, & Crain,

2005). This framework recommends that all students take the same, rich mathematics courses in K–8. The chapters describing high school pathways and data science set out a structure for high school that will be new to many administrators, including the provision of a pathway in data science and statistics that can be taken as an alternative, or in addition, to calculus. This pathway should be open to all students, not only those who have been selected as mathematically oriented in younger grades. The provision of real data, and the encouragement of students to ask their own questions of the data, has the potential to broaden participation and make Science, Technology, Engineering, and Mathematics (STEM) pathways considerably more equitable. As new courses are developed and introduced into schools, it is important that administrators hold equity as a guiding principle and work to encourage equitable participation in the new courses.

The instructional vignettes in the framework can guide administrators to develop an awareness of the different teaching strategies and classroom conversations that provide opportunities to improve professional practice, and reflect upon the ways they can nurture these types of experiences for their mathematics teachers. The vignettes highlight the central role of classroom discourse and rich, open tasks in teaching and learning mathematics. One key perspective for administrators to recognize is that standards-driven instruction does not mean that each task results in learning of a single standard—in fact, multiple standards can often be learned through engagement with the rich tasks with multiple access points called for in Chapter 2; and mastery-based assessment at the “big idea” level (as described in Chapter 11) helps to reinforce the experience of mathematics as a sense-making, relevant activity. Administrators who understand that exploring a big idea through a single, rich task that provides opportunities for students to communicate their thinking with their peers and their teacher also understand that this often results in multiple standards learned, or reconnected with, in ways that foster both positive disposition toward mathematics and learning that lasts.

Additionally, administrators must acknowledge the inequities often perpetuated through traditional assessment strategies in the mathematics classroom, and how these assessment approaches can be re-envisioned (as described in Chapter 10) to provide a balanced approach in assessing the effectiveness of mathematics instruction. They understand that the results of multiple assessment strategies—rather than a single score on a test—reflect a more complete understanding of student learning. Standards-based assessment provides an approach to grading that focuses learning on standards and mastery rather than emphasizing grade ranges or percentages. Broadened approaches to assessment in a district/school often mean that administrators prioritize participation in ongoing professional learning on the topic of mathematics education and assessment of learning. Administrators leverage their understanding and use of the Multi-Tiered System of Support (MTSS; https://www.cde.ca.gov/ci/cr/ri/mtsscomprti2.asp) by supporting teachers in aspects of MTSS implementation such as integration of instruction with intervention and a focus on continuous improvement.

Several ways that administrators can help support and incentivize effective professional learning are outlined in "Effective Teacher Professional Development" (Darling-Hammond, Hyler, & Gardner, 2017):

1. Since a critical component of rich learning is the planning time and pedagogical knowledge necessary to facilitate an active mathematics learning environment, administrators should prioritize time for professional learning and collaboration when designing schedules. Professional learning communities, peer coaching and observations across classrooms, and collaborative planning all provide important opportunities for educator learning.
2. Periodic needs assessments (at school or district level) use staff surveys to identify areas of professional learning most needed and desired by educators.

This helps ensure that professional learning is connected to practice and makes impact on practice much more likely.

3. District and school administrators should identify and develop expert teachers as mentors and coaches to support the professional learning of other educators. These “expert teachers” need their own support, structure, and professional learning in order to be effective.
4. Districts and schools should ensure that professional learning opportunities are integrated with efforts to implement legal requirements, such as the Every Student Succeeds Act (ESSA) school improvement initiatives. Mandates, such as the use of data to inform instruction and the creation of positive and inclusive learning environments, are primarily effective only when educators experience them as supportive of their improving classroom practice, as opposed to compliance exercises that add more paperwork to busy days.
5. In order to address professional learning needs of rural communities and to develop intra-district and intra-school collaboration, Titles II and IV of ESSA should be used to support technology-facilitated opportunities for professional learning and coaching.
6. District and school administrators can seek out funding which supports professional learning opportunities and connect this to continuing education units. These opportunities can include many of the types listed below, such as institutes, workshops, mathematics-specific conferences, and seminars, and also sustained engagement in collaboration, mentoring, and coaching. Possible funding sources include Local Control Accountability Plans, state and federal grant programs, community/business partnerships, and foundations.

Some specific tools to aid instructional leaders in supporting quality mathematics instruction include organizations that are available to partner with schools, as well as observation and planning guides:

- CMP, the California Mathematics Project (https://cmpso.org/), is a statewide network "dedicated to providing students a rich, rigorous, and coherent mathematics curriculum taught by competent and confident mathematics teachers who foster all students' proficiency in mathematics—achieving equity in quality." The regional sites of the CMP work with schools and districts to craft professional development appropriate to local needs.
- The Silicon Valley Mathematics Initiative (https://svmimac.org/) is an organization whose professional development offerings are available to member districts and schools throughout California.
- County offices of education (COEs) offer disciplinary and professional learning expertise that is often more than (especially small) districts can maintain, and COEs can work with districts to design and organize many professional learning opportunities.
- The 5x8 card (https://www.serpinstitute.org/5x8-card) is a tool designed by and for site administrators to guide their instructional observations to focus on student actions that lead to powerful mathematics learning.
- Inside Mathematics has collected tools for administrators (https://www.insidemathematics.org/tools-for-educators/tools-for-administrators). These include a tool for planning reflective conversations with teachers and guides for building and sustaining rigorous instructional programs.

These organizations and tools enable administrators' critical role in conveying high expectations for mathematics instruction—expectations made attainable by providing teachers with resources, including time for planning lessons, professional learning, and collaboration—with a focus on and aligned to agreed-upon school-wide priorities and strategies. Administrators can provide constructive, informative feedback that builds on teachers' strengths, while the teachers implement their plans. Frequent discussions about mathematics teaching and collaborations around mathematics lessons can allow the school administrator to engage teachers in productive conversations and provide

relevant feedback on instructional practices. The general observation pattern in many California schools—where a classroom teacher is observed formally once a year—is insufficient for educators to gain an understanding of, and support, teachers' instruction. Scheduling frequent and sustained interaction with teachers improves an administrator's engagement with students and teachers, and allows them to glean a more complete picture of the instructional practices used by their teachers and which supports are needed to bring about positive growth.

Role of Parents, Guardians, and Families

While the school classroom is a primary learning environment for mathematics education, home and community also play significant roles. Through involvement at every level, parents, guardians, and families can motivate students to develop a lifelong appreciation of mathematics learning. Families can also provide a supportive home setting for students to learn and prepare for school. Enlisting parents, guardians, and families in understanding and supporting authentic mathematics education and active learning pedagogy is key.

The passage below from *Black, Indigenous, and Latinx Parents as Partners in Mathematics Education* by TODOS: Mathematics for ALL (2020) provides insights about the assets parents bring when invited into the teaching and learning process:

> Black, Indigenous, and Latinx parents have a lot to offer classrooms; however, they are not always asked to join and be a part of the instruction. Ishimaru, Barajas-López, and Bang (2105) has argued for the involvement of parents from nondominant groups in schooling not as passive recipients of knowledge, but as "expert collaborators and fellow leaders." (p. 14). Given our current expectation of online and hybrid classes, schools can develop an online learning culture leveraging school/home connections that support mathematics identity and agency for students and parents. Research on Latinx parents visiting classrooms suggests that observations

and debriefs of classroom visits were one way that parents were able to both reflect on ways to support their students and develop leadership in mathematics education (Civil & Menéndez, 2012).

Because the CA CCSSM and this framework present mathematics instruction that is significantly different than what many parents experienced as students, it is critical to educate parents and guardians about what to expect and about the reasons and research behind the changes. Educating and engaging parents and guardians should include opportunities for them to experience rich, authentic, culturally-sustaining mathematical tasks in active-learning ways (including support for parents who speak languages other than English), not simply written descriptions of it. Validating and valuing parents', guardians', and families' central contributions to education is enhanced when they have opportunities to use their own language, culture, and knowledge through relevant experiences rooted in the school context.

Furthermore, parents and guardians who become more knowledgeable through such an experience can more effectively support students' learning beyond the classroom. Parents and guardians can monitor their student's progress not just for content knowledge, but for understanding of and engagement in mathematical practices or a developing inclination to use mathematics to make sense of their world. Parents and guardians can also foster social interactions (e.g., by providing support for collaborative classroom or out-of-classroom projects) and become involved in educational activities promoted at the school site (e.g., math fairs and math clubs).

A model to support the development of family and school partnerships is the National Parent Teacher Association (PTA), which has developed standards for Family-School Partnerships (https://www.cde.ca.gov/ci/sc/cf/ch12.asp#link11). These standards focus on several aspects of the partnership, providing recommendations on how to foster effective communication and trust to support students' success. In addition to the standards, the National PTA has developed a guide that provides a rubric with

examples for what family-school partnerships look like at the emerging, progressing, and excelling levels. Parents, guardians, families, and school leaders may want to use these examples to evaluate and enhance the family-school collaboration at their school site. Specifically, involving parents who have a background in mathematics (including in such areas as the building trades and cooking, as well as more traditional STEM areas) will help develop partnerships with the community that can provide much-needed support for classroom instruction.

The California *ELA/ELD Framework* (https://www.cde.ca.gov/ci/rl/cf/) provides specific suggestions for parent, guardian, and family involvement when those families speak a language other than English or are new to the United States. When possible, having parents who have experience with mathematics and speak a home language that students also speak would be a great support for the parents of those students who are not as experienced with mathematics (CDE 2014, Chapter 11).

Some specific resources to strengthen relationships with parents, guardians, and families include organizations that position them as partners in the education of their children and offer tools and resources:

- The National PTA, which has developed standards for Family-School Partnerships (https://www.cde.ca.gov/ci/sc/cf/ch12.asp#link11). These standards focus on several aspects of the partnership, providing recommendations on how to foster effective communication and trust to support students' success. In addition to the standards, the National PTA has developed a guide that provides a rubric with examples for what family-school partnerships look like at the emerging, progressing, and excelling levels. Parents, guardians, families, and school leaders may want to use these examples to evaluate and enhance the family-school collaboration at their school site. Specifically, involving parents who have a background in mathematics (including in such areas as the building trades and cooking, as well as more traditional STEM areas) will help develop

partnerships with the community that can provide much-needed support for classroom instruction.

- The Parent Institute for Quality Education (PIQE) (https://www.piqe.org/) is a national organization which originated in San Diego and is working mostly in California, with evidence-based programs that engage, empower and transform parents to actively engage in their children's education and strengthen parent-school collaboration. PIQE provides empowering information, skills development and support systems for low-income families, communities of color, emerging multicultural learner and immigrant families benefiting approximately 2.1 million children throughout its history.
- The Lawrence Hall of Science, the Science Center of the University of California, Berkeley (https://www.lawrencehallofscience.org/lawrence-at-home) offers a wide range of resources for families, including its flagship program FAMILY MATH.
- Beyond Teaching English: Supporting High School Completion by Immigrant and Refugee Students from the Migration Policy Institute (https://www.migrationpolicy.org/).
- Family Engagement and Support for Hybrid and Distance Learning from Californians Together (https://www.californianstogether.org/).
- Encouraging and Sustaining ELL Parent Engagement from ¡Colorín colorado! (https://www.colorincolorado.org/).
- From Abriendo Puertas (https://ap-od.org/):
    - Latino Parent Voices: What our Families Need Now for Education from Abriendo Puertas
    - Latino Parent Voices: What Our Families Need Now full report in English and in Spanish
    - Mini-Sessions for Parents and Caregivers in English and in Spanish
- From TODOS: Mathematics for ALL (2020) (https://www.todos-math.org/): Black, Indigenous, and Latinx Parents as Partners in Mathematics Education

Conclusion

A broad system of support to enable all students to succeed in their mathematics learning consists of many interconnected parts. Teachers, as the drivers of learning, continually refine and adapt their practice to address the many dimensions in creating a rich mathematical learning environment focused on active learning for all students in their classrooms. By supporting teachers with the resources, time, insight, and encouragement to become ever-more effective practitioners of their craft, administrators serve a critical role in the system. The elements for effective professional development described in this chapter provide administrators and other stakeholders with guidance on creating high-quality learning experiences for teachers, and the examples listed are a small sampling of the variety of professional development experiences available. Supporting teachers, both in their own learning and in their teaching, ultimately supports the students who rely upon these teachers.

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