Mathematics Framework
Chapter 1: Mathematics for All: Purpose, Understanding, and Connection

Second Field Review Draft

Introduction

A society without mathematical affection is like a city without concerts, parks, or museums. To miss out on mathematics is to live without an opportunity to play with beautiful ideas and see the world in a new light.

—Francis Su (2020)

Welcome to the 2021 *Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve* (*Mathematics Framework*). This framework serves as a guide to implementing the California Common Core State Standards for Mathematics (CA CCSSM or the Standards). Built upon underlying and updated principles of *focus*, *coherence*, and *rigor*, the Standards hold the promise of enabling all California students to become powerful users of mathematics in order to better understand and positively impact the world—in their careers, in college, and in civic life

Why Learn Mathematics?

Without mathematics, there's nothing you can do. Everything around you is mathematics. Everything around you is numbers.

—Shakuntala Devi, Author & "Human Calculator"

Mathematics grows out of curiosity about the world. Humans are born with an intuitive sense of numerical magnitude (Feigenson, Dehaene, and Spelke 2004), and this intuitive sense develops in early life into knowledge of number words, numerals, and the quantities they represent.

Give babies a set of blocks, and they will build and order them, fascinated by the ways the edges line up. Children will look up at the sky and be delighted by the V formations in which birds fly, or by the way the moon follows the same path as the sun. Count a set of objects with a young child, move the objects and count them again, and they will be enchanted by the fact they still have the same number (Boaler, 2019a). Human minds want to see and understand patterns (Devlin, 2006).

Mathematics engenders wonder. Just look around: bees use hexagons to build honeycombs, the number pi can be found in the shapes of rivers as they bend into loops, and seashells bring the Fibonacci sequence to life. Mathematics is full of wonder even outside of nature. Have you ever wondered what calculations were used to build the Pyramids? Or how suspension bridges work? What about innovations such as the moon landing or the birth of the Internet? Mathematics is at the heart of humanity and the natural world, though most of us did not get the chance to wonder mathematically in school. Too often, the joy and fascination young children's experiences with mathematics are replaced by dread and dislike when mathematics is introduced as a fixed set of methods to accept and remember.

Whether applied to everyday problem-solving, personal finances, community challenges, cooking, design, or a whole range of careers, learning mathematics enriches life. The ability to use mathematics flexibly and accurately also empowers people to influence their lives, communities, and the larger world in important ways. For example, algebra can help explain how quickly water can become contaminated and how many people drinking that water can become ill each year. Statistics and probability help us understand the risks of earthquakes and other such events, and can even predict what and how ideas spread.

Young students' work in mathematics is firmly rooted in their experiences in the world (Piaget and Cook, 1952). Numbers name quantities of objects or measurements such as time and distance, and operations such as addition and subtraction are represented by manipulations of such objects or measurements. Soon, the set of whole numbers itself becomes a context that is concrete enough for students to grow curious about and to reason within—with real-world and visual representations always available to support reasoning.

Students who use mathematics powerfully can maintain this connection between mathematical ideas and meaningful contexts. Historically, too many students lose the connection at some point between primary grades and graduation from high school. The resulting experience creates students who see mathematics as an exercise in memorized procedures that match different problem types. Yet, university professors and employers want graduates with critical thinking and reasoning skills. A robust understanding of mathematics, including data science, forms an essential component for many careers in the rapidly-changing and increasingly technology-oriented world of the twenty-first century.

This Framework takes the evidence-based stance that all students are capable of accessing and achieving success in school mathematics in the ways envisioned in California Common Core Standards for Mathematics (CA CCSSM). “Achieving success in school mathematics” means becoming inclined and able to consider novel situations (arising either within or outside mathematics) through a variety of appropriate mathematical tools. In turn, successful students can use those tools to understand the situation and, when desired, to exert their own power to affect the situation. Thus, mathematical power is not reserved for a few, but available to all.

Audience

The *Mathematics Framework* is intended to serve many different audiences, each of whom contributes to the shared mission of helping all students become powerful users of mathematics as envisioned in the CA CCSSM. First and foremost, the *Mathematics Framework* is written for teachers and those educators who have the most direct relationship with students around their developing proficiency in mathematics. As in every academic subject, developing powerful thinking requires contributions from many; and so this framework is also directed to:

- parents and caretakers of kindergarten through grade twelve (K–12) students who represent crucial partners in supporting their students’ mathematical success;
- designers and authors of curricular materials whose products help teachers to implement the standards through engaging, authentic classrooms;
- educators leading pre-service and teacher preparation programs whose students face a daunting but exciting challenge of preparing to engage students in meaningful, coherent mathematics;

- professional learning providers who can help teachers navigate deep mathematical and pedagogical questions as they strive to create coherent K–12 mathematical journeys for their students;
- instructional coaches and other key allies supporting teachers to improve students' experiences of mathematics;
- site, district, and county administrators who want to support improvement in mathematics experiences for their students;
- college and university instructors of California high school graduates who wish to use the framework in concert with the standards to understand the types of knowledge, skills, and mindsets about mathematics that they can expect of incoming students;
- educators focused on other disciplines so that they can see opportunities for supporting their discipline-specific instructional goals while simultaneously reinforcing relevant mathematics concepts and skills; and
- assessment writers who create curriculum, state, and national tests that signal which content is important and the determine ways students should engage in the content.

What Do We Know about How Students Learn Mathematics?

Students learn best when they are actively engaged in questioning, struggling, problem solving, reasoning, communicating, making connections, and explaining. The research is overwhelmingly clear that powerful mathematics classrooms thrive when students feel a sense of agency in mathematics (a willingness to engage in the discipline, based in a belief that they can make progress through productive struggle) and an understanding that the intellectual authority in mathematics rests in mathematical reasoning itself—in other words, that mathematics makes sense (Nasir, 2002; Gresalfi et al, 2009; Martin, 2009; Boaler & Staples, 2008). These factors support students as they develop their own identities as powerful mathematics learners and users. Further, active-learning experiences enable students to engage in a full range of mathematical activities—

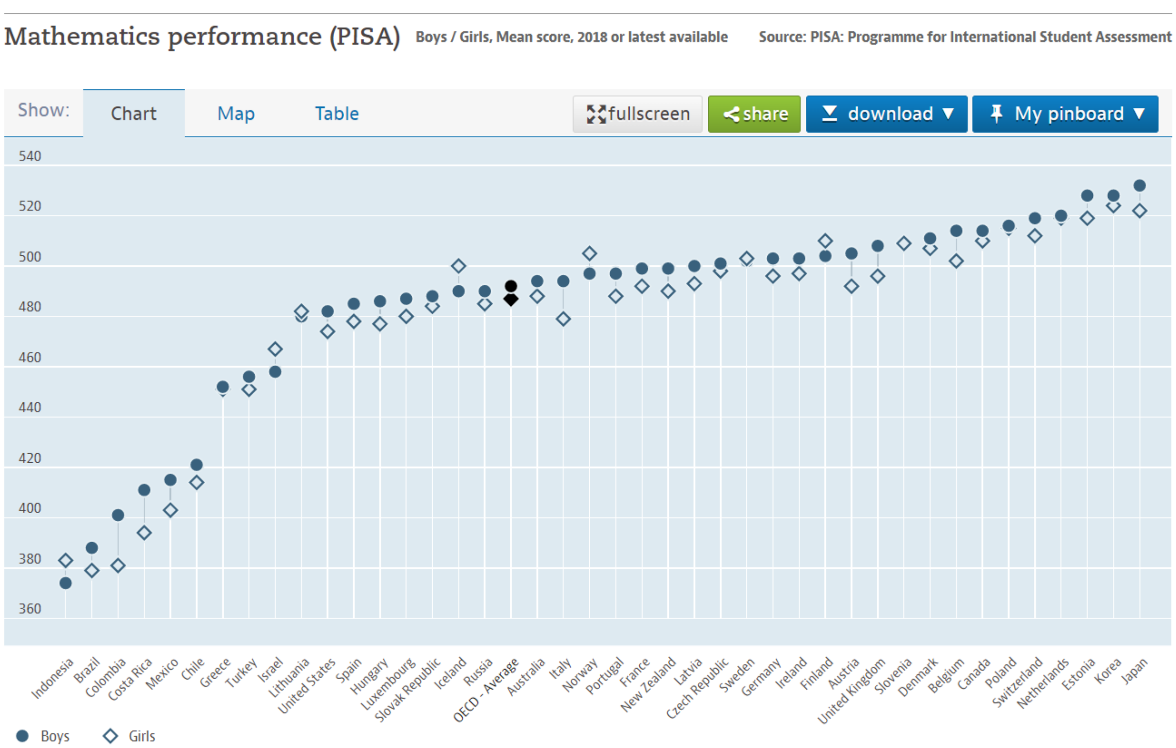
exploring, noticing, questioning, solving, justifying, explaining, representing and analyzing—making clear that mathematics represents far more than calculating.

Neuroscience research has shown that the highest achieving people have more interconnected brains—with different brain pathways communicating with each other (Menon, 2015; Kalb, 2017). This develops in students when they encounter mathematical ideas in different ways. In one example, Park and Brannon (2013) found that when students worked with numbers and also saw the numbers as visual objects, brain communication was enhanced and student achievement increased. A range of different research studies throughout K–12 have shown the importance of visual thinking in mathematics (West, 2004; Alibali and Nathan, 2012; Boaler, Chen, Williams, and Cordero, 2016; Boaler, 2019b). Researchers even found that after four 15-minute sessions of playing a game with a number line, differences in knowledge between students from low-income backgrounds and those from middle-income backgrounds were eliminated (Siegler and Ramani, 2008). The greatest brain development occurs when students are engaged in these multidimensional ways and they are given opportunities to struggle, to make mathematical connections, and to reason about ideas (Darling-Hammond et al., 2020).

All mathematical ideas can be considered in different ways—visually, through touch or movement, through building, modeling, writing and words, through apps, games and other digital interfaces, as well as through numbers and algorithms. Fingers have been shown to be particularly important as a visual and physical representation for students, enabling the development of important brain areas (Boaler, Chen, Williams, and Cordero, 2016). The tasks used in classrooms should encourage multi-dimensional forms of engagement. Tasks that offer multiple ways to engage with and represent mathematical ideas also support students with learning differences (Lambert and Sugita, 2016), as well as high achievers seeking greater challenges, often these are the same students (Freiman, 2018). The Universal Design for Learning (UDL) guidelines can support students with learning differences because they are designed to support learning for all (CAST, 2018).

However, these advances in what is known about how students learn mathematics have not been incorporated in US mathematics education in the ways they have been in many other high-achieving countries. As figure 1 shows, the US now ranks about 32nd in the world in mathematics on the Program for International Student Assessment (PISA), well below the OECD average – both as a function of how the United States teaches mathematics and how its systems have tolerated inequality in funding, staffing, and curriculum access. The CA CCSSM were based on research on how high-achieving countries organize and teach mathematics. In many other countries, standards guiding content in each grade are fewer, higher, and deeper, with greater coherence and integration, topics are studied more deeply, with applications to real world problems, use of mathematical practices, collaborative problem-solving strategies, untracked classrooms, and an integrated approach to high school mathematics. While the US developed the Common Core standards, there is still work to be done to reach the kind of curriculum organization and teaching that allows for consistently high achievement in mathematics.

Figure 1.0: Mathematics Performance (PISA)



Long description of this graphic

Source: Organization for Economic Co-operation and Development, 2021 (https://data.oecd.org/pisa/mathematics-performance-pisa.htm).

Mathematics as a Launchpad or a Gatekeeper?

> Math literacy and economic access are how we are going to give hope to the young generation.
>
> —Bob Moses and Charles Cobb (Moses and Cobb, 2002, 12)

Mathematics can serve as a powerful launchpad into nearly any career or course of study, and research is also clear that *all* students are capable of becoming powerful mathematics learners and users (Boaler, 2019b, c). In high-achieving countries, the vast majority of students of all backgrounds achieve high levels of success in mathematics: there is very little variation across schools or student groups. However, the notion that success in mathematics can be widespread runs counter to many adults' and students' ideas about school mathematics in the United States. Most adults can recall times when they received messages during their school or college years that they were not cut out for mathematics-based fields. Negative messages, either explicit ("I think you'd be happier if you didn't take that hard mathematics class") or implicit ("I'm just not a math person"), both imply that only some people can succeed. Perceptions can also be personal ("Math just doesn't seem to be your strength") or general ("This test isn't showing me that these students have what it takes in math. My other class aced this test."). And they can also be linked to labels ("low kids," "bubble kids," "slow kids") that lead to a differentiated and unjust mathematics education for students.

Further, the race-, class-, and gender-based differences among those who pursue more advanced mathematics make it clear that messages students receive about who belongs in mathematics are often associated with race, socioeconomic status, language, and gender. For example, in 2015, Sarah-Jane Leslie, Andrei Cimpian, and colleagues interviewed university professors in different subject areas to gauge their perceptions that people need special intellectual "gifts" to be successful in a particular field. The results were staggering: the more prevalent the idea, in any academic field, the fewer women and people of color participating in that field. This outcome held across all thirty subjects in the study. More mathematics professors believed that students needed a special ability than any other professor of Science, Technology, Engineering, and Mathematics (STEM) content. The study highlights the subtle ways that students may be dissuaded from continuing in mathematics and underscores the important role mathematics teachers play in communicating messages that mathematics success is only achievable for select students. This is despite evidence of mathematical excellence being achieved by all student groups (NCSM and TODOS, 2016).

Students internalize negative messages. Before students have opportunities to excel, many self-select out of mathematics when exposed repeatedly to them, and when mathematics is experienced as the memorization of meaningless formulas—perhaps because they see no relevance for their learning, and no longer recognize the inherent value or purpose in learning mathematics. When mathematics is organized differently and pathways are opened to all students, mathematics plays an important role in students' lives, propelling them to quantitative futures and rewarding careers (Burdman et al., 2018; Guha et al., 2018; Getz et al., 2016; Daro and Asturias, 2019).

An aim of this framework is to respond to the structural barriers put in the place of mathematics success: equity influences all aspects of this document. Some overarching principles that guide work towards equity in mathematics include the following:

- All students deserve powerful mathematics; high-level mathematics achievement is not dependent on rare natural gifts, but rather can be cultivated (Leslie et al., 2015; Boaler, 2019 a, b; Ellenberg, 2014).
- Access to an engaging and humanizing education—a socio-cultural, human endeavor—is a universal right.
- Student engagement must be a design goal of mathematics curriculum design, co-equal with content goals.

- Students’ cultural backgrounds, experiences, and language are resources for learning mathematics (González, Moll, and Amanti, 2006; Turner and Celedón-Pattichis, 2011; Moschkovich, 2013).
- All students, regardless of background, language of origin, differences, or foundational knowledge are capable and deserving of depth of understanding and engagement in rich mathematics tasks.

In order to reach the goals of deep, active learning of mathematics for all, this framework is centered around the investigation of big ideas in mathematics taught in multi-dimensional ways that meet varied learning needs. This is a tall order. The framework draws on research from the past two decades in mathematics education, neuroscience, and the psychology of learning that begin to illuminate an exciting path forward. The framework invites readers to reimagine mathematics and move toward a new century of mathematical excellence for all, innovation, and democratic participation.

Seeing Opportunities for Growth

> Hard work and persistence is more important for success in mathematics than natural ability. Actually, I would give this advice to anyone working in any field, but it’s especially important in mathematics and physics where the traditional view was that natural ability was the primary factor in success.
>
> —Maria Klawe, Mathematician, Harvey Mudd President (in Williams, 2018)

Fixed notions about student ability have led to considerable inequities in mathematics education. Particularly damaging is the idea of the “math brain” (Heyman, 2008)—that people are either born with a brain that is suited for math or not, in which case they should expect little success. Technologies that have emerged in the last few decades allowing researchers to understand the mind and brain have completely challenged this idea. With current technology, scientists can study learning in mathematics through brain activity; they can look at growth and degeneration and see the impact of different emotional conditions on brain activity. This work has shown—resoundingly—that almost all people possess the capacity to learn mathematics to very deep levels. Multiple studies have shown the incredible capacity of brains to grow and change within a short period of time in response to careful teaching (Huber et al., 2018; Luculano et al., 2015; Abiola and Dhindsa, 2011; Maguire, Woollett, and Spiers, 2006; Woollett and Maguire, 2011). Learning allows brains to form, strengthen, or connect brain pathways in a process of almost constant change and adaptation (Doidge, 2007; Boaler, 2019 b, c). An important goal of this framework is to recognize that every student is on a growth pathway and is capable of developing deep mathematical understanding and creativity.

The neuroscientific evidence that shows the potential of all students to reach high levels in mathematics is related to the evidence base that supports the importance of mindset messages. Stanford University psychologist Carol Dweck and her colleagues have conducted research studies in different subjects and fields for decades showing that people’s beliefs about personal potential changes the ways their brains operate and influences what they achieve. One of the important studies Dweck and her colleagues conducted took place in mathematics classes at Columbia University (Carr et al., 2012), where researchers found that young women received messaging that they did not belong in the discipline. When students with a fixed mindset—that is, a view that intelligence is innate and fixed or unchangeable—heard the message that mathematics was not for women, they dropped out. Those with a growth mindset, however, protected by the belief that anyone can learn anything with effort, ultimately rejected the stereotype and persisted. Dweck and her colleagues have shown, through multiple studies, that students with a growth mindset achieve at higher levels in mathematics, and that when students change their mindsets, from fixed to growth, their mathematics achievement increases (Blackwell, Trzesniewski, and Dweck, 2007; Dweck, 2008).

Another idea related to the “math brain” that teachers should challenge comes from social comparison. Students often believe that brains must be fixed, because some people appear to get ideas faster and to be naturally good at certain subjects. What these students do not realize is that brains grow and change every day. Each moment is an opportunity for brain growth and development and some students have developed stronger pathways on a different timeline. Teachers should strive to reinforce the idea that all students can develop those pathways at any time if they take the right approach to learning.

It is important for teachers to share the science of brain growth and clarify the idea that, although students are all unique, anyone can learn the content that is being taught, and productive learning and brain development is in part due to their effortful thinking. This understanding can be particularly effective at the beginning of the school year or mathematics course. Students may find the message liberating, and allow it to override any prevailing messaging that success in mathematics can only be achieved by a few students. When students learn about brain growth and mindset, they realize something critically important—no matter where they are in their learning, they can improve and eventually excel (Blackwell, Trzesniewski, and Dweck, 2007; Dweck, 2008).

Teachers should also underscore the importance and value of times of struggle. The importance of struggle has been shown through both brain-based and behavior-based studies. Daniel Coyle (2009), for example, studied the highest achieving people in different fields of work and found a characteristic shared by these achievers was a willingness to struggle—to work “at the edge of their understanding,” to make mistakes, correct them, move on, and create more. This, he found, was the optimal approach to accelerate learning.

This understanding is bolstered by brain-based studies. For instance, psychologist Jason Moser and his colleagues found that when young adults were performing simple tasks, those with growth mindsets exhibited stronger brain activity associated with adaptive responses to mistakes than did those with fixed mindsets. Indeed, those with growth mindsets performed better following a mistake than following correct answers, whereas those with fixed mindsets performed no better following a mistake. In the test used for the study, participants were able to identify their own errors quickly, and a growth mindset was associated with “heightened awareness of and attention to errors” (Moser et al., 2011). This fits into a range of neuroscientific work showing that times of struggle are productive for brains as they are the times when pathways are developing and strengthening.

This evidence becomes particularly important when considering that often when students struggle in mathematics class, they decide they do not have a "math brain" and give up. It is important for teachers to share the research on the benefits of struggle and encourage students to persevere when it seems easier to give up.

Meeting Varied Learning Needs

Once an educator recognizes and believes that every student can learn meaningful, grade-level mathematics to deep levels, the challenge is to create classroom experiences that allow all students to access mathematical thinking and persevere through challenges. In order to achieve access, students must begin from whatever past experiences and understandings they bring into the activity, and their perseverance must extend through (and perhaps beyond) the activity's target mathematical practice and content goals.

Creating such classroom experiences is not easy. Educators sometimes hear "differentiated instruction" as a requirement to create separate individualized plans and activities for each student, and despair at the scale of the task. In the approach outlined in Chapter 2 (Teaching for Equity and Engagement), a diversity of backgrounds and perspectives in classrooms is instead used as an asset which can launch deep exploration for all students engaged in exploring the same context and open problem.

This Framework's Chapter 2 (Teaching for Equity and Engagement) lays out components (and examples) of classroom instruction that can meet the needs of students in classrooms that are diverse in so many ways. The five components are

1. Plan Teaching Around Big Ideas
2. Use Open, Engaging Tasks
3. Teach Towards Social Justice
4. Invite Student Questions and Conjectures
5. Center Reasoning and Justification

Planning teaching around Big Ideas as defined here helps to enact the other four components.

Big ideas are central to the learning of mathematics, link numerous mathematics understandings into a coherent whole, and provide focal points for students' investigations (Charles, 2005). These big ideas and the connections among them comprise a "schema"— a map of the intellectual territory— that supports conceptual understanding. Learning scientists find that people learn more effectively when they understand a map of the domain and how the big ideas fit together (National Research Council, 2000). They can then locate facts and details within this schema and see how they fit together.

Big ideas are implemented in the classroom by developing major content goals through student investigations in authentic contexts, in ways that utilize and reinforce the Standards for Mathematical Practice (see Designing for Coherence, Focus, and Rigor, below).

In addition to more accurately reflecting the real-world practice and use of mathematics, instruction designed around big ideas allows a very broad range of student needs to be met: All students are able to access mathematical thinking thanks to familiar contexts ("low floor" investigations), students' experiences and linguistic and cultural stores are called on through student-generated questions and conjectures, and open tasks ensure that all students continue to find challenge by exploring as deeply as time allows ("high ceiling" investigations).

Multi-dimensional Mathematics

Another meaningful result from studies of the brain is the importance of brain connections. Vinod Menon (2015) and a team of researchers at Stanford University have studied the interacting networks in the brain, particularly focusing on the ways the brain works when it is solving problems—including mathematics problems. They found that even when people are engaged with a simple arithmetic question, five different areas of the brain are involved, two of which are visual pathways. The dorsal visual pathway is the main brain region for representing quantity.

Menon and other neuroscientists also found that communication between the different brain areas enhances learning and performance. Researchers Joonkoo Park and Elizabeth Brannon (2013) reported that different areas of the brain were involved when people worked with symbols, such as numerals, than when they worked with visual and spatial information, such as an array of dots. The researchers also found that mathematics learning and performance were optimized when these two areas of the brain were communicating with each other. Learning mathematical ideas comes not only through numbers, but also through words, visuals, models, algorithms, multiple representations, tables, and graphs; from moving and touching; and from other representations. But when learning reflects the use of two or more of these means and the different areas of the brain responsible for each communicate with each other, the learning experience improves.

For this reason, this framework highlights examples that are multi-dimensional, with mathematical experiences that are visual, physical, numerical, and more. These approaches align with the principles of UDL, a framework designed to help all students by making learning more accessible by encouraging the teaching of subjects through multiple forms of engagement, representation and expression. Visual and physical representations of mathematics are not only for young children, nor are they merely a prelude to abstraction or higher-level mathematics (Boaler, Chen, Williams, and Cordero, 2016). Some of the most important high-level mathematical work and thinking—such as the work of Fields medal winner Maryam Mirzakhani—is visual.

The different areas of neuroscientific research with evidence showing the potential of brains to grow and change, the importance of times of struggle, and the value in engaging with mathematics in multi-dimensional ways, should be shared with students. When messages such as these were shown in a free online class offered through a randomized controlled trial, students significantly increased their mathematics engagement in class and improved later achievement (Boaler, Dieckmann, Pérez-Núñez, Sun, and Williams, 2018).

Updating Coherence, Focus, and Rigor

The CA CCSSM were adopted by the State Board of Education in 2010 and modified in 2013. Over a decade of experience has made evident the kinds of challenges the Standards posed for teachers, administrators, curriculum developers, professional learning providers, and others.

Because the standards were then new to California educators (and to curriculum writers), the 2013 California *Mathematics Framework* was comprehensive in its treatment of the content standards, including descriptions and examples for most individual standards. In the intervening years, many more sources of examples, exemplars, and models of sample tasks illustrating the understanding and know-how intended by each standard have emerged. Thus, the need is different in 2021: The framework no longer needs to provide as much expansion on the individual standards; rather, curriculum designers and California educators need guidance for creating mathematics experiences that provide access to the coherent body of understanding and strategies of the discipline. Standards are explored within the context of learning progressions across (or occasionally within) grades, rather than one standard at a time.

When the standards and the subsequent framework were each adopted, they both reflected an approach based on identifying major and minor standards—a recognition that it can be difficult for teachers to address all standards while maintaining a rich and deep learning experience for all students. This approach essentially eliminated key areas of content (such as data literacy). This framework reflects a revised approach, advocating that publishers and teachers avoid organizing around the detailed content standards, and instead organize around the most important mathematical ideas—"big ideas"—that most often connect *many* standards in a more coherent whole. While important standards previously identified as “major” or “power” standards will continue to be very prominent, the framework encourages addressing them in the context of progressions and big ideas.

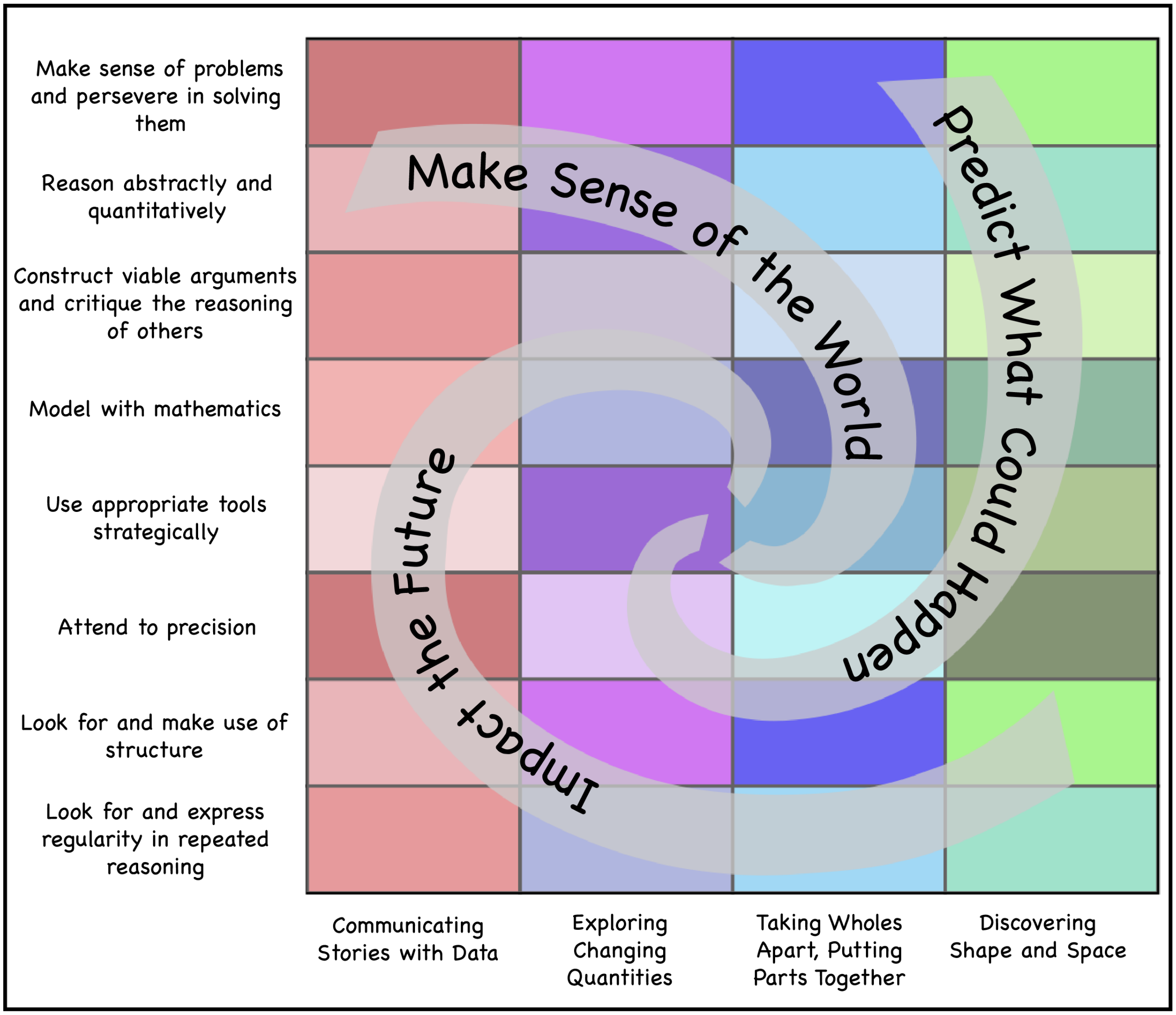
Designing for Coherence, Focus and Rigor: Drivers of Investigation and Content Connections

With the goal of motivating students to learn coherent, focused, and rigorous mathematics, this framework identifies three **Drivers of Investigation** (DIs), which provide the “why” of learning mathematics, to pair with four categories of **Content Connections** (CCs), which provide the “how and what” mathematics (CA CCSSM) is to be learned in an activity. Together with the Standards for Mathematical Practice, the Drivers of Investigation are meant to propel the learning of the ideas and actions framed in the Content Connections in ways that are coherent, focused, and rigorous.

Standards for Mathematical Practice The "How"	Content Connections The "What"	Drivers of Investigation The "Why"
Students will… SMP1. Make Sense of Problems and Persevere in Solving them SMP2. Reason Abstractly and Quantitatively SMP3. Construct Viable Arguments and Critique the Reasoning of Others SMP4. Model with Mathematics SMP5. Use Appropriate Tools Strategically SMP6. Attend to Precision SMP7. Look for and Make Use of Structure SMP8. Look for and Express Regularity in Repeated Reasoning	**while…** CC1. Communicating Stories with Data CC2. Exploring Changing Quantities CC3. Taking Wholes Apart, Putting Parts Together CC4. Discovering Shape and Space	**in order to…** DI1. Make Sense of the World (Understand and Explain) DI2. Predict What Could Happen (Predict) DI3. Impact the Future (Affect)

The following diagram is meant to illustrate the ways that the Drivers of Investigation relate to the Content Connections and Standards for Mathematical Practice. Note that any Driver of Investigation can go with any of the Content Connections and any of the Standards for Mathematical Practice.

Figure 1.1: Content Connections, Mathematical Practices, and Drivers of Investigation



[Long description of this graphic]

Drivers of Investigation

The Content Connections should be developed through investigation of questions in authentic contexts; these investigations will naturally fall into one or more of the following Drivers of Investigation. The DIs are meant to serve a purpose similar to that of the Crosscutting Concepts in the California Next Generation Science Standards, as unifying reasons that both elicit curiosity and provide the motivation for deeply engaging with authentic mathematics. The aim of the Drivers of Investigation is to ensure that there is always a reason to care about mathematical work, and that investigations allow students to make sense, predict, and/or affect the world. The DIs are:

- DI1: Make Sense of the World (Understand and Explain)
- DI2: Predict What Could Happen (Predict)
- DI3: Impact the Future (Affect)

Used in conjunction with the Content Connections, and the Standards for Mathematical Practice, the Drivers of Investigation can guide instructional design. For example, students can make sense of the world (DI1) by exploring changing quantities (CC2) through classroom discussions wherein students have opportunities to construct viable arguments and critique the reasoning of others (SMP.3).

Teachers can use the DIs to frame questions or activities at the outset for the class period, the week, or longer; or refer to these in the middle of an investigation (perhaps in response to the "Why are we doing this again?"-type questions students often ask), or circle back to these at the conclusion of an activity to help students see "why it all matters." Their purpose is to leverage students' innate wonder about the world, the future of the world, and their role in that future, in order to motivate productive inclinations (the SMPs) that foster deeper understandings of fundamental ideas (the CCs and the Standards), and to develop the perspective that mathematics is a lively, flexible endeavor by which we can appreciate and understand so much of the inner workings of our world.

Content Connections

The four Content Connections described in the framework organize content and provide mathematical coherence through the grades:

- CC1: Communicating Stories with Data
- CC2: Exploring Changing Quantities
- CC3: Taking Wholes Apart, Putting Parts Together
- CC4: Discovering Shape and Space

Content Connection 1: Communicating Stories with Data

With data all around us, even the youngest learners make sense of the world through data—including data about measurable attributes. In grades TK–5, students describe and compare measurable attributes, classify objects, count the number of objects in each category, represent their discoveries graphically, and interpret the results. In grades 6–8, prominence is given to statistical understanding, reasoning with and about data, reflecting the growing importance of data as the source of most mathematical situations that students will encounter in their lives. In grades 9–12, reasoning about and with data is emphasized, reflecting the growing importance of data as the source of most mathematical situations that students will encounter in their lives. Investigations in a data-driven context—data either generated/collected by students, or accessed from publicly-available sources—help maintain and build the integration of mathematics with students' lives (and with other disciplines such as science and social studies). Most investigations in this category also involve aspects of CC2: Exploring Changing Quantities.

Content Connection 2: Exploring Changing Quantities

Young learners' explorations of changing quantities support their development of meaning for operations, and types of numbers. The understanding of fractions established in TK–5 provides students with the foundation they need to explore ratios, rates, and percents in grades 6–8. In grades 9–12, students make sense of, keep track of, and connect a wide range of quantities, and find ways to represent the relationships between these quantities in order to make sense of and model complex situations.

Content Connection 3: Taking Wholes Apart, Putting Parts Together

Students engage in many experiences with taking apart quantities and putting parts together strategically, including utilizing place value in performing operations (such as making 10), decomposing shapes into simpler shapes and vice versa, and relying upon unit fractions as the building blocks of whole and mixed numbers. This Content

Connection also serves as a vehicle for student exploration of larger-scale problems and projects, many of which will intersect with other CCs as well. Investigations in this CC will require students to decompose challenges into manageable pieces, and assemble understanding of smaller parts into understanding of a larger whole.

Content Connection 4: Discovering Shape and Space

In the early grades, students learn to describe their world using geometric ideas (e.g., shape, orientation, spatial relations). They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes, thus setting the stage for measurement and initial understanding of properties such as congruence and symmetry. Shape and space work in grades 6–8 is largely about connecting foundational ideas of area, perimeter, angles, and volume notions to each other, to students’ lives, and to other areas of mathematics, such as nets and surface area or two-dimensional shapes to coordinate geometry. In grades 9–12, the CA CCSSM supports visual thinking by defining congruence and similarity in terms of dilations and rigid motions of the plane, and through its emphasis on physical models, transparencies, and geometry software.

Coherence

> I like crossing the imaginary boundaries people set up between different fields—it's very refreshing. There are lots of tools, and you don't know which one would work. It's about being optimistic and trying to connect things.
>
> —Maryam Mirzakhani, Mathematician, 2014 Fields Medalist

The Standards for Mathematical Practice (SMPs) and the Content Standards are intended to be equally important in planning, curriculum, and instruction (CA CCSSM, 2013, 3). The content standards, however, are far more detailed at each grade level, and are more familiar to most educators; as a result, the content standards continue to provide the organizing structure for most curriculum and instruction. Because the content standards are more granular, curriculum developers and teachers find it easy when designing lessons to begin with one or two content standards and choose tasks and activities which develop that standard. Too often, this reinforces the concept as an isolated idea.

Instructional materials should primarily involve tasks that invite students to *make sense* of these big ideas, *elicit wondering* in authentic contexts, and *necessitate mathematical investigation*. The National Research Council and the Commission on Behavioral and Social Sciences summarized research on the optimum ways to learn and concluded that: “Superficial coverage of all topics in a subject area must be replaced with in-depth coverage of fewer topics that allows key concepts in the discipline to be understood. The goal of coverage need not be abandoned entirely, of course. But there must be a sufficient number of cases of in-depth study to allow students to grasp the defining concepts in specific domains within a discipline.” (Bransford, Brown, and Cocking, 2000, 20). Drawing from this research, this framework recommends teaching to ‘big ideas’ that allow teachers and students to explore ‘key concepts’ in depth, through investigations. Big ideas in mathematics are central to the learning of mathematics, link numerous mathematical understandings into a coherent whole, and provide focal points for students’ investigations (Charles and Carmel, 2005). The value of focusing on big ideas for teachers, and their students, cannot be overstated. Voices in the field emphasize this: “When teachers work on identifying and discussing big ideas, they become attuned to the mathematics that is most important and that they may see in tasks, they also develop a greater appreciation of the connections that run between tasks and ideas” (Boaler, Munson, and Williams, 2018). In each grade band section (TK–2, 3–5, 6–8, 9–12), the description focuses on several big ideas that have great impact on students’ conceptual understanding of numbers, and which encompass multiple content standards.

An authentic activity or problem is one in which students investigate or struggle with situations or questions about which they actually wonder. Lesson design should be built to elicit that wondering. Many contexts can support student questioning—wondering—about mathematical questions. These include those related to students’ lives (culturally relevant contexts), more mundane everyday contexts, and also purely mathematical contexts, when students have enough experience to notice patterns and wonder within them. Authenticity of context and task is as much a factor of classroom implementation as it is of the context itself. Student curiosity can be provoked in these contexts and many more:

- Environmental observations and issues on campus and in their local community (concurrently helping students develop their understanding of California's Environmental Principles and Concepts)
- Puzzles
- Patterns—numerical or visual—in purely mathematical settings
- Real-world or fictional contexts in which something happens or changes over time

This framework sets out these organizing ideas to provide *coherence* and to help teachers avoid losing the forest for the trees. That is, discrete content standard success does not necessarily assemble in students' minds into a coherent big-picture view of mathematics.

This framework's responses to the challenge posed by the principle of coherence are: designing instruction around big ideas that link Standards for Mathematical Practice (how we do mathematics) with Content Connections (both within and across domains, see section below) and Drivers of Investigation (why we do mathematics); progressions of learning across grades (thus, grade-band chapters rather than individual grade chapters); and relevance to students' lives. Principles guiding grade-band chapters include

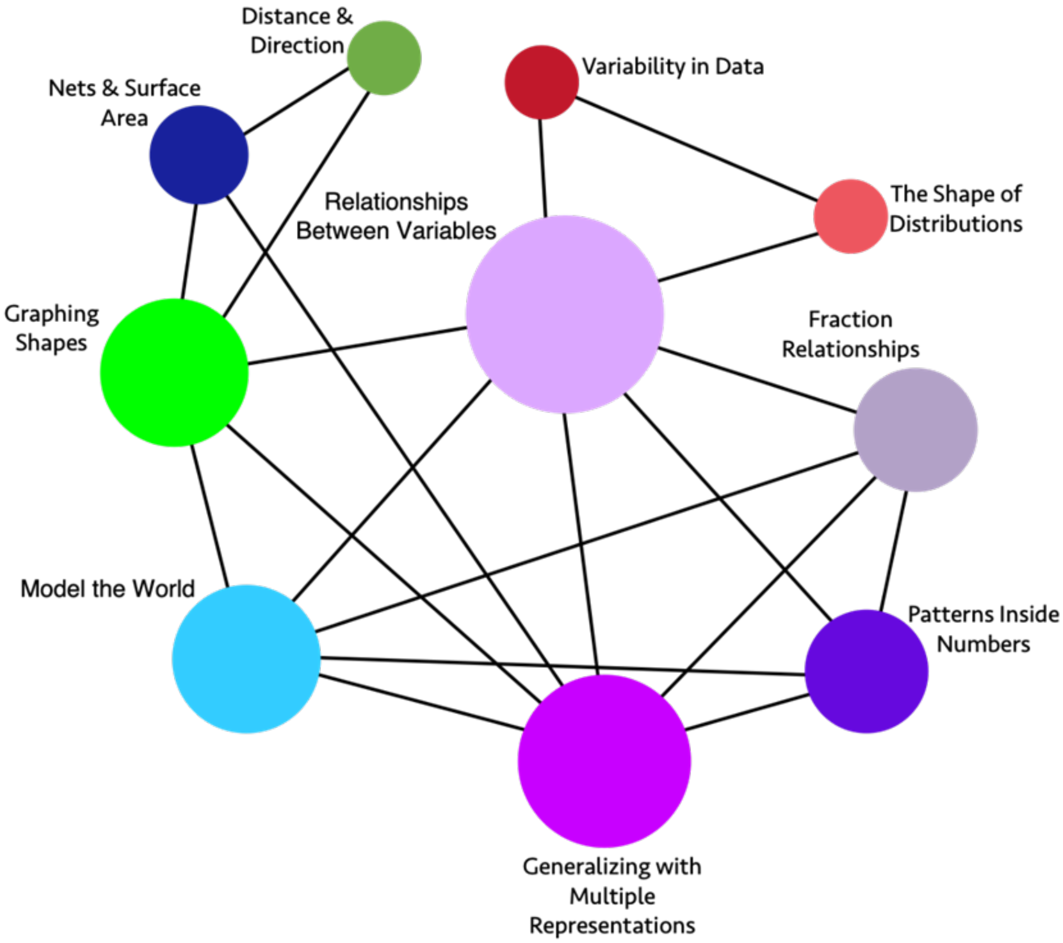
- design from a smaller set of big ideas, spanning TK–12, within each grade band;
- a preponderance of student time spent on authentic problems that engage multiple content and practice standards situated within one or more big ideas;
- design for connections: between students' lives and mathematical ideas and strategies, and between different mathematical ideas; and

- constant attention to opportunities for students to bring other aspects of their lives into the mathematics classroom: How does this mathematical way of looking at this phenomenon compare with other ways to look at it? What problems do you see in our community that we might analyze? Teachers who relate aspects of mathematics to students’ cultures often achieve more equitable outcomes (Hammond, 2014).

Figure 1.2. Below is the Grade 6 network map, which identifies the Big Ideas of the grade level mathematics and highlights the connections between them. The sizes of the circles in the diagram vary to give an indication of the relative importance of the topics. The connecting lines between circles show links among topics, and are suggestive of ways to design instruction so that multiple topics are addressed simultaneously.

Big ideas network maps for each grade are included within the grade level-banded chapters.

Figure 1.2: Grade 6 Big Ideas



[Link to long description of the graphic]

Focus

> I didn't want to just know the names of things. I remember really wanting to know how it all worked.
>
> —Elizabeth Blackburn, Winner of the 2009 Nobel Prize for Physiology or Medicine.

The principle of *focus* is closely tied to the goal of *depth* of understanding. The principle derives from a need to confront the mile-wide but inch-deep mathematics curriculum experienced by many.

Instructional design built on moving from one content standard to the next underscores the challenging reality that the standards simply contain *too many* concepts and strategies to address comprehensively in this manner. Teachers often opt to choose between covering standards at an adequate depth (while skipping some topics), or including all topics from the standards for their grade level and compromising opportunities to reach rich, deep understandings.

One common approach to the coverage-versus-depth challenge is to designate some content standards more important than others. An unintentional result of this, in many schools, is that the standards deemed “less important” simply are not addressed.

The standards, however, are *not* a design for instruction, and should not be used as such. The standards lay out expected understanding and know-how at the grade levels, and expected mathematical practices at the conclusion of high school. They say little about how to achieve that understanding and know-how or build those practices. By analogy to baking a cake, the standards tell us what the cake should look, smell, taste, and feel like at the end (and at intermediate points along the way), but do not give guidance on activities or ingredients that produce those characteristics.

This framework’s answer to the coverage-vs-depth challenge posed by the principle of *focus* is to lay out principles for (and examples of) instructional design that make the Standards achievable. These principles include the following:

- Focus on investigations and connections, not individual standards: class activities should be designed around big ideas, and typically should necessitate several clusters of content standards and multiple practice standards, as part of an investigation (there may be some occasions when a single content standard is essentially synonymous with a big idea). Connections between those content standards then becomes an integral part of the class activity, and not an additional topic to cover. The twin focus on investigations and connections is reflected in titles and structure of the grade-banded chapters, Chapters 6, 7, and 8, as well as in the DIs and CCs.

- Focus on the ways in which activities fit within a multi-year progression of learning. Educators must understand how each student experience extends earlier ideas (including those from prior years), and what future understanding will draw on current learning. Students must experience mathematics as coherent across grades. The focus in the framework on progressions across years (in chapters 3, 4, and 5 as well as in the grade-band chapters 6, 7, and 8) reflects this focus. This is in contrast to an approach of choosing “power standards;” rather, focus on critical progressions.
- Construct tasks that are worthy of student engagement.
    - Problems (tasks which students do not already have the tools to solve) *precede* teaching of the focal mathematics which are necessitated by the problem. That is, the major point of a problem is to raise questions that can be answered, and promote students using their intuition, before learning new mathematical ideas (Deslauriers, McCarty, Miller, Callaghan, and Kestin, 2019).
    - Exercises (tasks which students already have the tools to solve) should either be embedded in a larger problem which is motivating (an authentic problem as above, perhaps involving patterns, games, or real-world contexts such as environmental or social justice), or should address strategies whose improvement will help students accomplish some motivating goal.
    - Students should learn to see their goal as investigating mathematical ideas, asking important questions, making conjectures and developing curiosity about mathematics and mathematical connections.

Rigor

> True rigor is productive, being distinguished in this from another rigor which is purely formal and tiresome, casting a shadow over the problems it touches.
>
> —Émile Picard (1905)

In this framework, *rigor* refers to an integrated way in which conceptual understanding, strategies for problem-solving and computation, and applications are learned, so that each supports the other. This definition is more specific and somewhat more demanding than the CA CCSSM's requirement that "*rigor* requires that conceptual understanding, procedural skill and fluency, and application be approached with equal intensity" (CA CCSSM, 2013, 2). For a fuller exploration of the meaning of rigor in mathematics and its implications for instruction, see (Dana Center, 2019).

This definition expresses the basis of mathematical rigor: reasoning which enables understanding "all the way down to the bottom" (Ellenberg 2014, 48), often expressed in terms of validity and soundness of arguments. According to the definition used here, conceptual understanding cannot be considered rigorous if it cannot be *used* to analyze a novel situation encountered in the world; computational speed and accuracy cannot be called rigorous unless it is accompanied by conceptual understanding of the strategy being used, including why it is appropriate in a given situation; and a correct answer to an application problem is not rigorous if the solver cannot explain to the client both the ideas of the model used and the methods of calculation.

In particular, **rigor is *not* about abstraction**. In fact, a push for premature abstraction leads, for many students, to an absence of rigor in the sense used in this framework. It is true that more advanced mathematics often occurs in more abstract contexts. This leads many to value more abstract subject matter as a marker of rigor. "Abstraction" in this case usually means "less connected to reality."

But mathematical abstraction is in fact *deeply* connected to reality: When second graders use a representation with blocks to argue that the sum of two odd numbers is even, in a way that other students can see would work for *any* two odd numbers (a representation-based proof; see Schifter, 2010), they have *abstracted* the idea of odd number, and they know that what they say about an odd number applies to one, three, five, etc. (Such an argument reflects SMP.7: Look for and make use of structure.)

Abstraction must grow out of experiences in which students experience the same mathematical ideas and representations showing up and being useful in different contexts. When students figure out the size of a population, after 50 months, with a growth of three percent a month; their bank balance after 50 years if they can earn three percent interest per year; and the number of people after 50 days who have contracted a disease that is spreading at three percent per day, they will abstract the notion of a quantity growing by a certain percentage per time period, and recognize that they can use the same reasoning in each case to understand the changing quantity.

Rigorous mathematics learning as defined here can occur through an investigation-driven learning cycle. Notice in this brief description that the application to an authentic context supports the development of mathematical concepts and problem-solving strategies.

- Exploration in a familiar context generates authentic questions and predictions or guesses
- Attempts to understand those questions reveals mathematical objects, quantities, and relationships
- Mathematical concepts and strategies are developed and/or introduced to understand these objects, quantities, and relationships
- Mathematical work is translated back to the original context and compared with predictions and reasonableness

Thus, the challenge posed by the principle of *rigor* is to provide all students with experiences that interweave concepts, problem-solving (including appropriate computation), and application, such that each supports the other. To meet this challenge, the *Mathematics Framework* emphasizes these principles for designing instruction:

- Abstract formulations should *follow* experiences with multiple contexts that call forth similar mathematical models.
- Contexts for problem-solving should be chosen to provide representations for important concepts, so that students may later use those contexts to reason about the mathematical concepts raised. The Drivers of Investigation provide broad reasons to think rigorously ("all the way to the bottom") in ways that linkages between and through topics (Content Connections) are recognized, valued and internalized.
- Computation should serve a genuine need for students to know, typically in a problem-solving or application context. In particular, in order for computational algorithms (standard or otherwise) to be understood rigorously, students must be able to connect them to conceptual understanding (via a variety of representations, as appropriate) and be able to use them to solve authentic problems in diverse contexts. An important aspect of this understanding is to recognize the power that algorithms give in problem solving: knowing only single-digit multiplication and addition facts, it is possible to compute any sum, difference, or product involving whole numbers or (finite-length) decimals.
- Applications should be authentic to students and should be enacted in a way that requires students to explain or present solution paths and alternate ideas.

Assessing for Coherence, Focus and Rigor

> Mathematical notation no more is mathematics than musical notation is music. A page of sheet music represents a piece of music, but the notation and the music are not the same; the music itself happens when the notes on the page are sung or performed on a musical instrument. It is in its performance that the music comes alive; it exists not on the page but in our minds. The same is true for mathematics.
>
> —Keith Devlin (2003)

In order to gauge what students know and can do in mathematics, there is a need to broaden assessment beyond narrow tests of procedural knowledge to better capture the connections between content and SMPs. For example, assessing a good mathematical explanation includes how students mathematize a problem, connect the mathematics to the context, and explain their thinking in a clear, logical manner that leads to a conclusion or solution (Callahan et al., 2020). Helpful mathematics guidelines from the English Learner Success Forum (ELSF) center on focus area five, assessment of mathematical content, practices, and language. Specifically, these guidelines note the need to capture and measure students' progress over time (ELSF guideline 14), and attend to student language produced (ELSF guideline 15).

New to this Framework

The 2013 edition of the *California Mathematics Framework* included detailed explications and examples of most content standards, recognizing the substantial shift represented by the 2010 adoption of the CA CCSSM. Educators in California now have developed strong understanding of the individual standards, aided substantially by the 2013 framework. To address the needs of California educators in 2022, this 2022 edition of the framework includes several new emphases and types of chapters.

To raise the profile of the imperative for TK–12 mathematics instruction to foster more equitable outcomes in mathematics and science, the framework offers Chapter 2: Teaching for Equity and Engagement, which promotes instruction that fosters equitable learning experiences for all, and challenges the deeply-entrenched policies and practices that lead to inequitable outcomes. This single chapter replaces two separate chapters—one on instruction and one on access—in the previous framework. While some people argue for a false dichotomy between equity and high achievement, this framework rejects that notion in favor of emphasizing ways good teaching leads to equitable and higher outcomes. Instruction and equity together create instructional designs that can bring about equitable outcomes. The state-level commitment to equity extends throughout the framework, and every chapter highlights considerations and approaches designed to help mathematics educators create and maintain equitable opportunities for all.

Given the more-advanced understanding of the individual standards among educators, this framework focuses on connections between standards and across grades. Two chapters are devoted to exploring the development, across the TK–12 grade timeframe, of particular content areas. One such area is number sense across TK–12 (Chapter 3: Number Sense), a crucial foundation for all later mathematics and early predictor of mathematical perseverance. The other is data science (Chapter 5: Data Science, TK–12), which has become tremendously important in the field since the last framework. The other new chapter, Chapter 4: Exploring, Discovering, and Reasoning With and About Mathematics, presents the development of a related cluster of SMPs across the entire TK–12 timeframe. While it is beyond the scope of the *Mathematics Framework* to develop such a “progression” for all SMPs, this chapter can guide the careful work that is required to develop SMPs across the grades.

The idea of learning progressions across multiple grade levels is emphasized further in the grade-banded chapters, Chapter 6: Mathematics: Investigating and Connecting, Transitional Kindergarten through Grade Five; Chapter 7: Mathematics: Investigating and Connecting, Grades Six through Eight; and Chapter 8: Mathematics: Investigating and Connecting, High School. The Big Ideas for each grade band, in the form of overarching Drivers of Investigation and Content Connections, provide a structure for promoting relevant and authentic activities for students, sample tasks, snapshots, and vignettes to illustrate the building of ideas across grades. “The key to prioritizing learning is to move beyond grade-level check lists and instead think of progressions of important learning that cut across grade levels” (CGCS, 2020).

Chapter 9: Structuring School Experience for Equity and Engagement, and Chapter 10: Supporting Educators in Offering Equitable and Engaging Mathematics Instruction, present guidance designed to build an effective system of support for teachers as they facilitate learning for their students; it includes advice for administrators and leaders and sets out models for effective teacher learning. Chapter 11: Technology and Distance

Learning in the Teaching of Mathematics, describes the purpose of technology in the learning of mathematics, introduces overarching principles meant to guide such technology use, and general guidance for distance learning. Chapter 12: Assessment in the 21st Century, addresses the need to broaden assessment practices beyond answer finding to record student thinking, and to create assessment systems that emphasize growth of leaning over performance. The chapter reviews “Assessment for Learning” and concludes with a brief overview of the Common Core-aligned standardized assessment used in California: the California Assessment of Student Performance and Progress. Chapter 13: Instructional Materials to Support Equitable and Engaging Learning of the California Common Core State Standards for Mathematics, is intended to support publishers and developers of instructional materials to serve California’s diverse student population. This chapter provides guidance for local districts on the adoption of instructional materials for students in grades 9–12, the social content review process, supplemental instructional materials, and accessible instructional materials. Chapter 14: Glossary: Acronyms, Terms, and Tables, provides a list of commonly used acronyms, definitions for many of the mathematics education terms frequently used in this framework, and some helpful tables of operation situations.

Explicit Focus on Environmental Principles and Concepts (EP&Cs). While the Drivers of Investigations and Content Connections are fundamental to the design and implementation of this framework and the standards, teachers must be mindful of other considerations that are a high priority for California’s education system, including the EP&Cs, which allow students to examine issues of environmental and social justice.

Environmental literacy is championed by the California Department of Education, the California Environmental Protection Agency, and the California Natural Resources Agency. It is also fully embraced in a 2015 report prepared by a task force of the State Superintendent of Public Instruction, *A Blueprint for Environmental Literacy: Educating Every Student in, about, and for the Environment* (CDE Foundation, 2015). Strongly reinforcing the goal of environmental literacy for all kindergarten through grade twelve students, the blueprint states that “the central approach for achieving environmental literacy…is to integrate environmental literacy efforts into California’s increasingly coherent and aligned K–12 education landscape so that all teachers are given the opportunity to use the environment as context for teaching their core subjects.” It also advocates that all teachers have the opportunity to use the environment as a relevant and engaging context to “provide learning experiences that are culturally relevant” for teaching their core subjects of math, English language arts, English language development, science, and history–social science.

The Environmental Principles and Concepts (Figure 1.3) are the critical understandings that California has identified for every student in the state to learn and be able to apply. Developed in 2004, California’s EP&Cs reflect the fact that people, as well as their cultures and societies, depend on Earth’s natural systems. The underlying goal of the EP&Cs is to help students understand the connections between people and the natural world so that they can better assess and mitigate the consequences of human activity.

Figure 1.3: California’s Environmental Principles (CEEI, 2020)

Principle	Description
Principle I—People Depend on Natural Systems	The continuation and health of individual human lives and of human communities and societies depend on the health of the natural systems that provide essential goods and ecosystem services.
Principle II—People Influence Natural Systems	The long-term functioning and health of terrestrial, freshwater, coastal and marine ecosystems are influenced by their relationships with human society.
Principle III—Natural Systems Change in Ways that People Benefit from and Influence	Natural systems proceed through cycles that humans depend upon, benefit from, and can alter.

Principle	Description
Principle IV—There are no Permanent or Impermeable Boundaries that Prevent Matter from Flowing Between Systems	The exchange of matter between natural systems and human societies affects the long-term functioning of both.
Principle V—Decisions Affecting Resources and Natural Systems are Complex and Involve Many Factors	Decisions affecting resources and natural systems are based on a wide range of considerations and decision-making processes.

Classroom activities can simultaneously introduce the EP&Cs and develop important mathematics through investigations into students' local community and environment. The EP&Cs and environmental literacy can provide meaningful ways to teach and amplify many of the ideas that are embedded in the CA CCSSM (Lieberman, 2013). Vignettes and snapshots that provide examples of connections between mathematics instruction and the EP&Cs are included in Chapters 5, 6, 7, and 8 of this framework.

Every Californian needs to be ready to address the environmental challenges of today and the future, take steps to reduce the impacts of natural and anthropogenic (human-made) hazards, and act in a responsible and sustainable manner with the natural systems that support all life. As a result, the EP&Cs have become an important piece of the curricular expectations for all California students in mathematics and other content areas.

Long Descriptions of Graphics for Chapter 1

Figure 1.0: Mathematics Performance (PISA)

Boys / Girls, Mean score, 2018 or latest available.

Source: Programme for International Student Assessment (PISA)

Location	Boys	Girls
Australia	494	488
Austria	505	492
Belgium	514	502
Brazil	388	379
Canada	514	510
Chile	421	414
Colombia	401	381
Costa Rica	411	394
Czech Republic	501	498
Denmark	511	507
Estonia	528	519
Finland	504	510
France	499	492
Germany	503	496
Greece	452	451
Hungary	486	477
Iceland	490	500
Indonesia	374	383
Ireland	503	497
Israel	458	467
Italy	494	479
Japan	532	522
Korea	528	524
Latvia	500	493
Lithuania	480	482
Luxembourg	487	480
Mexico	415	403
Netherlands	520	519
New Zealand	499	490
Norway	497	505
OECD - Average	492	487
Poland	516	515
Portugal	497	488
Russia	490	485
Slovak Republic	488	484
Slovenia	509	509
Spain	485	478
Sweden	502	503
Switzerland	519	512
Turkey	456	451
United Kingdom	508	496
United States	482	474

Return to graphic.

Figure 1.1: Content Connections, Mathematical Practices, and Drivers of Investigation

Three Drivers of Investigation (DIs) provide the “why” of learning mathematics: Make Sense of the World (Understand and Explain); Predict What Could Happen (Predict); Impact the Future (Affect). The DIs overlay and pair with four categories of Content Connections (CCs), which provide the “how and what” mathematics (CA-CCSSM) is to be learned in an activity: Communicating Stories with Data; Exploring Changing Quantities; Taking Wholes Apart, Putting Parts Together; Discovering Shape and Space. The DIs work with the Standards for Mathematical Practice to propel the learning of the ideas and actions framed in the CCs in ways that are coherent, focused, and rigorous. The Standards for Mathematical Practice are: Make sense of problems and persevere in solving them; Reason abstractly and quantitatively; Construct viable arguments and critique the reasoning of others; Model with mathematics; Use appropriate tools strategically; Attend to precision; Look for and make use of structure; Look for and express regularity in repeated reasoning. Return to graphic.

Figure 1.2: Grade 6 Big Ideas

The graphic illustrates the connections and relationships of some sixth-grade mathematics concepts. Direct connections include:

- Variability in Data directly connects to: The Shape of Distributions, Relationships Between Variables
- The Shape of Distributions directly connects to: Relationships Between Variables, Variability in Data
- Fraction Relationships directly connects to: Patterns Inside Numbers, Generalizing with Multiple Representations, Model the World, Relationships Between Variables
- Patterns Inside Numbers directly connects to: Fraction Relationships, Generalizing with Multiple Representations, Model the World, Relationships Between Variables
- Generalizing with Multiple Representations directly connects to: Patterns Inside Numbers, Fraction Relationships, Model the World, Relationships Between Variables, Nets & Surface Area, Graphing Shapes

- Model the World directly connects to: Fraction Relationships, Relationships Between Variables, Patterns Inside Numbers, Generalizing with Multiple Representations, Graphing Shapes
- Graphing Shapes directly connects to: Model the World, Generalizing with Multiple Representations, Relationships Between Variables, Distance & Direction, Nets & Surface
- Nets & Surface directly connects to: Graphing Shapes, Generalizing with Multiple Representations, Distance & Direction
- Distance & Direction directly connects to: Graphing Shapes, Nets & Surface Area
- Relationships Between Variables directly connects to: Variability in Data, The Shape of Distributions, Fraction Relationships, Patterns Inside Numbers, Generalizing with Multiple Representations, Model the World, Graphing Shapes

Return to graphic.