Mathematics Framework
Chapter 14—Glossary: Acronyms, Terms, and Tables

Second Field Review Draft

Introduction

This chapter provides a list of acronyms commonly used in mathematics teaching and learning conversations, followed by working definitions and descriptions for many of the terms in this framework. Some terms are defined in the chapters with their initial use in the framework; those definitions are repeated here. Note that many of the mathematical terms used in this document carry multiple interpretations; as such, teachers are encouraged to rely on the mathematical definitions provided in the curricula adopted by their local educational agencies.

Acronyms

Acronym	Full Title or Term
CAASPP	California Assessment of Student Performance and Progress
CACCSSM	California Common Core State Standards for Mathematics
CC	Content Connection
DL	Distance Learning
DI	Driver of Investigation
MIC	Mathematics: Investigating and Connecting
ELA	English Language Arts
ELD	English Language Development
SBE	State Board of Education
SMP	Standard for Mathematical Practice
UDL	Universal Design for Learning

Terms

Acute angle. An angle with a measure of less than 90 degrees.

Additive reasoning. Adding or subtracting to solve various problems indicates additive reasoning. By joining, comparing, and separating quantities, children engage in additive reasoning. In upper elementary grades, however, additive reasoning can be mistakenly applied to ratio problems. For example, adding the same quantity to both A and to B in the ratio A:B will change the ratio between the quantities. Multiplicative reasoning is the extension of additive reasoning which enables productive strategies when working with ratios.

Algebra. The part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations.

Algorithm. A step-by-step method of calculating.

Area. In geometry, the area can be defined as the space occupied by a flat shape or the surface of an object. The area of a figure is the number of unit squares that cover the surface of a closed figure.

Array/Area Models of multiplication. In an array, discrete objects are arranged in rows, forming a rectangle; the number of rows represents one factor of a multiplicative situation and the quantity in each row represents the second factor. The area model is a continuous view of multiplication. The problem is pictured as a rectangle, the dimensions of which represent the factors being multiplied. Example: 18 x 35 would be visualized as a rectangle with the shorter sides of length 18 units, and the longer sides of length 35 units.

Attributes. Characteristics or qualities by which to describe and distinguish objects or geometric figures.

Authentic. Authentic describes a problem, activity, or context in which students investigate or struggle with situations or questions about which they actually wonder. Lesson design should be built to elicit that wondering. For example, environmental observations and issues on campus and in the local community provide rich contexts for student investigations and mathematical analysis as they concurrently help students develop their understanding of California's Environmental Principles and Concepts.

In contrast, an activity is inauthentic if students recognize it as a straightforward practice of recently-learned techniques or procedures, including the repackaging of standard exercises in forced real-world contexts. Mathematical patterns and puzzles can be more authentic than such real-world settings.

Benchmark fraction. A familiar, well-understood fraction, commonly used to position other fractions on a number line or to compare numbers. Examples: 1/2, 3/4, 5/5.

Big Idea. Big ideas in math are central to the learning of mathematics, link numerous math understandings into a coherent whole, and provide focal points for students' investigations.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each athlete on a sports team's roster.

Calculus. The branch of mathematics that deals with the finding and properties of derivatives and integrals of functions, by methods originally based on the summation of infinitesimal differences. The two main types are *differential calculus* and *integral calculus*.

Cardinality. An understanding of how numbers are ordered, and how to count accurately, matching a number name to the quantity counted.

Categorical variable. Categorical variables are any variables where the data represent groups, such as eye color or favorite food.

Coherence. A unified understanding of topics in and related to mathematics. This framework answers the challenge posed by the principle of coherence by: focusing on big ideas, both as Drivers of Investigation (the reasons why we do mathematics), and Content Connections (both within and across domains); progressions of learning across grades (thus, grade-band chapters rather than individual grade chapters); and relevance to students' lives.

Complex Fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Comparison model of multiplication. A multiplication situation which calls for thinking about "how many times as much" one quantity is than another. This interpretation of multiplication is introduced in grade four. Example: interpreting 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5.

Compose. To put numbers or geometric figures together strategically and purposefully, typically to simplify calculation or to recognize properties.

Computational algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

Conceptual understanding. Refers to an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which is it useful. They have organized their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they already know. Conceptual understanding also supports retention. Because facts and methods learned with understanding are connected, they are easier to remember and use, and they can be reconstructed when forgotten (Source: Adding It Up, 2001).

Confidence interval. A range of values likely to include a population value with a certain degree of confidence.

Conjecture. A proposed statement before it has been proven or justified.

Content Connections. Content themes that provide mathematical coherence through the grades. Content Connections include: CC1: Communicating Stories with Data, CC2: Exploring Changing Quantities, CC3: Taking Wholes Apart, Putting Parts Together, and CC4: Discovering Shape and Space.

Culturally relevant pedagogy. A theoretical model that not only addresses student achievement but also helps students to accept and affirm their cultural identity while developing critical perspectives that challenge inequities that they and others in their lives have experienced (Ladson-Billings, 1995a). It is a pedagogy that empowers students intellectually, socially, emotionally, and politically by using cultural referents to impart knowledge, skills, and attitudes (Ladson-Billings, 1994). It rests on three criteria: (a) students must experience academic success, (b) students must develop and/or maintain cultural competence, and (c) students must develop a critical consciousness through which they challenge the status quo of the current social order (Ladson-Billings, 1995b).

Culturally responsive teaching. An approach that leverages the strengths that students of color bring to the classroom to make learning more relevant and effective (see Gay, 2002, 2018). A major goal of culturally responsive teaching is to reverse patterns of underachievement for students of color. Culturally responsive teaching requires teachers to recognize the cultural capital and tools that students of color bring to the classroom and to utilize their students' cultural learning tools throughout instruction.

Culturally sustaining pedagogy. Affirms and respects the key components of culturally relevant pedagogy and culturally responsive teaching that preceded it, but also takes them to the next level (see Paris, 2012). Instead of just accepting or affirming the backgrounds of students of color as seen in culturally relevant pedagogy; or connecting to students' cultural knowledge, prior experiences, and frames of reference as we see in culturally responsive pedagogy; culturally sustaining pedagogy views schools as places where the cultural ways of being in communities of color are sustained and developed, rather than eradicated. Culturally sustaining pedagogy promotes equality across racial and ethnic communities and seeks to ensure access and opportunity. Culturally sustaining pedagogy also supports students to critique and question dominant power structures in societies.

Data literacy. The ability to reason with and about data, to make good decisions based on data, to ask questions of data, and to use statistical reasoning.

Data science. An emerging discipline that includes understanding principles of data collection, data manipulation, data analysis, inference, and interpretation and communication.

Decompose. To take numbers or geometric figures apart strategically and purposefully, typically to simplify calculation or to recognize properties.

Double number line diagram. A diagram in which two number lines subdivided in the same way are set one on top of the other with zeros aligned. Although the number lines are subdivided in the same way, the units in each may be different, which allows for the illustration of ratio relationships. Double number lines can also be constructed vertically.

Drivers of Investigation. Unifying reasons that both elicit curiosity and provide the motivation for deeply engaging with authentic mathematics.

Designated English Language Development (Designated ELD). Instruction provided during a protected time in the regular school day for focused instruction on the state-adopted ELD standards. During Designated ELD, English learners develop critical English language skills necessary for accessing academic content in English. (Title 5 *California Code of Regulations* [5 *CCR*] Section 11300[a]).

Distance learning. Instruction in which pupils and instructor are in different locations and pupils are under the general supervision of a certificated employee of the local educational agency.

Efficient. Refers to methods of calculation that are economical in terms of time and the simplicity of calculation steps.

Emerging English learner student. English learners at this level have limited receptive and productive English skills. These students can engage in cognitively demanding activities when provided substantial linguistic support. (CDE, 2012, 20).

English learner (EL): English learners are those students for whom there is a report of a primary language other than English and who, on the basis of the state approved language proficiency assessment (grades transitional kindergarten through grade twelve), do not meet the state's definition of English proficiency (per *California Education Code* 313). They are students for whom language, culture, and literacy are valuable assets. (Adapted from the Coalition for English Learner Equity) See also **linguistically and culturally diverse students**.

Environmental Principles and Concepts (EP&Cs). The California EP&Cs are focused on the connections between humans and the natural world. They prepare students to address the environmental challenges of today and of the future, to mitigate and prepare for natural hazards, and to interact in a responsible and sustainable manner with the natural systems that support all life. The State Board of Education officially adopted the EP&Cs in 2004 making them an important piece of the curricular expectations for all California students.

Equal-groups model of multiplication. Modeling multiplication with objects or quantities in equal sized groups. The number of groups represents one factor of a multiplicative situation and the quantity in each group represents the second factor.

Equilateral. A geometric figure with sides all of equal length.

Equity. Equity refers to fairness in education rather than sameness. Drawing from Gutierrez (2012), equity includes four dimensions in mathematics education: (1) Access to tangible resources; (2) Participation in quality mathematics classes and success in them; (3) Student identity development in mathematics; and (4) Attention to relations of power.

Euler's formula. A mathematical formula in complex analysis that establishes the fundamental relationship between the trigonometric functions and the complex exponential function.

Expanded form. A way of writing a number, separating place values to show the value of each digit. Example: 4,256 = 4000 + 200 + 50 + 6.

Exponential function. A mathematical function in which an independent variable appears in one of the exponents.

Factor. One of the numbers being multiplied in a multiplication situation.

Fixed mindset. In a fixed mindset, people believe their basic qualities, like their intelligence or talent, are simply fixed traits.

Flexible. Numerical thinking and reasoning that is varied, strategic, and intentional. Examples of flexible use of number include: taking numbers apart by place value, adjusting numbers to make calculation easier; applying mathematical properties strategically.

Fluency. The ability to select and flexibly use appropriate strategies to explore and solve problems in mathematics.

Focus. Instruction should focus deeply on only those concepts that are emphasized in the standards so that students can gain strong foundational conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems inside and outside the mathematics classroom.

Focus. The depth of understanding about specific topics and concepts. This Framework addresses focus by emphasizing the need for activities to target big ideas that necessitate understanding of multiple content and practice standards, emphasizing connections between topics that allow for deeper exploration, and use of tasks that are worthy of sustained student engagement.

Fraction. A number expressible in the form *a/b* where *a* is a whole number and *b* is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.)

Function. A set of ordered pairs where each element from the first set (an input) is paired with exactly one element from the second set (an output). Functions can be expressed in a variety of ways, such as function notation (f(x) = …), sets of ordered pairs, graphs, and tables.

Generalized number. The practice of using a letter for a non-specific, general number. An early conception for a variable held by students.

Geometry. A branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, surfaces, and solids.

Growth mindset. In a growth mindset, people believe that their most basic abilities can be developed through dedication and hard work—brains and talent are just the starting point.

Hundreds chart. An array of the numbers 1 through 100, organized in 10 rows of 10, useful in developing understanding of counting, cardinality, the base ten number system, patterns.

Inferential statistics. The branch of statistics that generalizes about a population using data from a sample.

Integer. A number expressible in the form *a* or –*a* for some whole number *a.*

Integrated English Language Development (Integrated ELD). Instruction in which the California ELD Standards are used in tandem with the state-adopted academic content standards (5 *CCR* Section 11300[c]).

Integrated. Refers to both the connecting of mathematics with students' lives and their perspectives on the world, and to the connecting of mathematical concepts to each other. Integrated tasks, activities, projects, and problems are those which invite students to engage in both of these aspects of integration.

Irregular shapes. Shapes that have sides and angles of any length and size.

Isosceles. A type of geometric figure, such as a triangle or trapezoid, in which two side lengths are equal.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.

Linear relationships. A statistical term used to describe a straight-line relationship between two variables. Linear relationships can be expressed either in a graphical format or as a mathematical equation of the form y = mx + b.

Linguistically and culturally diverse students. A heterogeneous group of learners that includes students learning in Dual Language contexts, students who are multilingual, and students who have typically been labeled as English learners. These are students for whom language, culture, and literacy are valuable assets. (Adapted from the Coalition for English Learner Equity). See also **English learners** and Chapter 1.

Low-floor/high-ceiling task. A task that has an entry point that is accessible for all learners, regardless of math knowledge level, and is open-ended enough to allow learners to continue working toward ideas for a sustained length of time.

Manipulatives. Any of various objects or materials that students can touch and move around in order to help them learn mathematical and other concepts. Where physical objects are unavailable, **virtual manipulatives** may be a viable option.

Mastery based grading. Mastery based grading describes a form of grading that focuses on mastery of ideas, rather than points or scores. It communicates the mathematics students are learning, and students receive feedback on the mathematics they have learned or are learning, rather than a score. This helps students view their learning as a process that they can improve on over time, rather than a score or a grade that they often perceive as a measure of their worth.

Measurement division. See quotitive division.

Measures of variability. Describe how similar or varied the set of observed values are for a particular variable (data item). Measures of variability include the range, quartiles and the interquartile range, variance, mean absolute deviation and standard deviation.

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 20.

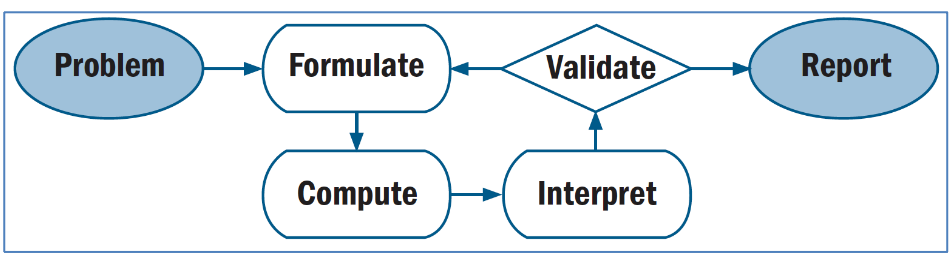
Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 10, 12, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Mode. The most frequently occurring value in a set of numerical data values.

Model/Modeling. “Modeling,” as used in the CA CCSSM is primarily about using mathematics to describe the world. In elementary mathematics, a model might be a representation such as a math drawing or a situation equation (operations and algebraic thinking), line plot, picture graph, or bar graph (measurement), or building made of blocks (geometry). In grades six through seven, a model could be a table or plotted line (ratio and proportional reasoning) or box plot, scatter plot, or histogram (statistics and probability). In grade eight, students begin to use functions to model relationships between quantities. In high school, modeling becomes more complex, building on what students have learned in kindergarten through grade eight. Representations such as tables or scatter plots are often intermediate steps rather than the models themselves.

Modeling Cycle.



Multiple. A product which is a whole number times another number is said to be a multiple. For example, 6 is a multiple of 2 since 2*3 = 6., and $5\sqrt{2}$ is a multiple of $\sqrt{2}$.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 x 4/3 = 4/3 x 3/4 = 1.

Multiplicative reasoning. The use of multiplication to solve problems is known as multiplicative reasoning. It is commonly used when solving ratio problems. For example, scaling up of a ratio to solve a proportional problem, such as “If sugar to flour is in the ratio of 2 parts to 5 parts, then how much sugar is needed for 15 flour parts?” Multiplicative reasoning would involve recognizing that since triple the amount of flour parts was needed (15 = 5*3), then triple the amount of sugar would be needed (6 = 2*3).

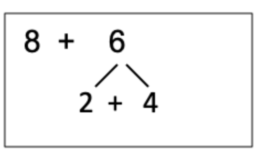
Multiplicative relationships. Where two quantities can be expressed as multiples of each other.

Necessitate. An activity or task necessitates a mathematical idea or strategy if the attempt to understand the situation or task creates for students a need to understand or use the mathematical idea or strategy.

Neuroscientific. Scientific study of the nervous system.

Non-standard units of measurement. Objects, such as small cubes, pens, paper clips, or other classroom materials that are used for making comparisons of length or other measurement, most commonly in the primary grades.

Number bond diagram.



An illustration showing how a student decomposes a number in order to calculate. Example, to add 8 + 6, the student decomposes 6 as 2 + 4, adds 8 + 2 and then adds the remaining 4.

Number line. A linear representation of a set of numbers.

Number path. A number path is a counting model used in primary grades where rectangles or other shapes are arranged in a path. Number paths can serve as a precursor to using number lines.

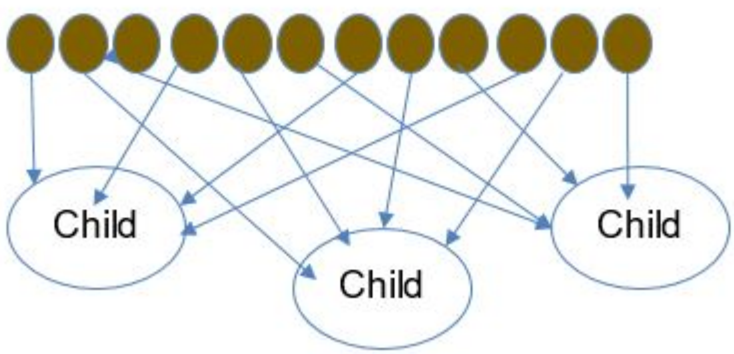
Number sense. Refers to an intuitive understanding of numbers, their magnitude, relationships, and how they are affected by operations.

Number talks/number strings. Short class discussions in which students solve a math problem mentally, share their strategies aloud, and determine a correct solution.

One-to-one correspondence. If each object in set A is paired with exactly one object from set B, and each object in B is paired with exactly one object in A, then the sets are said to be in one-to-one correspondence. This is often used in counting objects, by pairing a set with the counting numbers (1, 2, 3, ...).

Parallel. Lines in the same plane that never intersect.

Participation. Engaging with one's own ideas and the ideas of others (from Webb et al., 2014). Partitive Division Illustration of 12 cookies, shared among three children.



Partitive division. A division situation in which the divisor represents the number of equal groups and the quotient is the size of or quantity in each of the equal groups. This is also known as equal-sharing or, informally, "divvy-up" division. For example, the answer to "If there are 12 cookies and 3 children, and the cookies are shared equally, how many does each child receive?" is the quotient 12 ÷ 3 = 4, which indicates that each child receives 4 cookies. Since each child is considered a group, the quotient is the quantity per group and thus partitive. (See illustration of quotitive division for contrast.)

Perpendicular. Lines in the same plane that intersect at a right angle.

Place value structure. The value represented by a digit in a number on the basis of its position in the number.

Polyhedron. A three-dimensional shape with flat polygonal faces, straight edges and sharp corners or vertices.

Polynomial. An expression of more than two algebraic terms, especially the sum of several terms that contain different powers of the same variable(s).

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Probability model. Used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1.

Product. The result of a multiplication. Example: 12 is the product of 3 times 4.

Proofs by contradiction. A form of proof that establishes the truth or the validity of a proposition, by showing that assuming the proposition to be false leads to a contradiction.

Proofs by induction. A form of proof that allows you to prove a statement about an arbitrary number n by first proving it is true when n is 1 and then assuming it is true for $n=k$ and showing it is true for $n=k+1$.

Proportion. (a) Another term for a fraction of a whole. Example: The “proportion of the population that prefers product A” might be 60 percent. (b) A statement of equality between two ratios. Example: 4/8 = 1/2 or 4:8 =1:2 or “4 is to 8 as 1 is to 2.”

Proportional relationship. A collection of pairs of numbers that are in equivalent ratios. A ratio determines a proportional relationship—namely, the collection of pairs (ca,cb) for c positive. A proportional relationship is described by an equation of the form y=kx, where k is a positive constant (often called a constant of proportionality). (Source: Progressions for the Common Core State Standards in Mathematics [draft]. Grades 7–8, high-school Geometry, 2019)

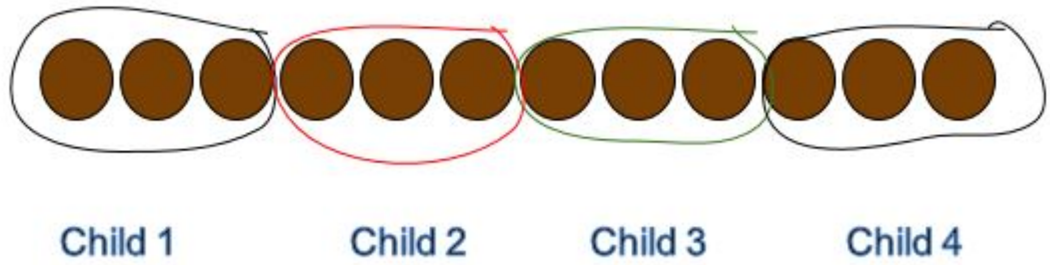
Properties of operations. There are four basic properties of real numbers: namely—commutative, associative, distributive, and identity. These properties only apply to the operations of addition and multiplication. That means subtraction and division do not have these properties built in.

Pythagorean Theorem. A theorem attributed to Pythagoras that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

Quadratic expression. An expression involving a squared term, e.g., x^2 + 1, or a product term, e.g., 3xy – 2x + 1.

Quantitative variables. Any variables where the data represent amounts (e.g., length, weight, or volume).

Quotitive division. (Also known as measurement division or repeated subtraction division) A division situation in which the divisor represents the size of or quantity in each of the equal groups, and the quotient tells the number of equal groups that can be formed. For example, the answer to the question, “If there are 12 cookies and each child is to receive 3 cookies, then how many children receive cookies?” is the quotient 12 ÷ 3 = 4, which indicates that 4 children received 3 cookies each. Since each child is a group, and the quotient is the number of groups, this is an example of a quotitive division problem. Quotitive Division Illustration of 12 cookies apportioned to three children, with each child getting three cookies:



Random sampling. A smaller group of people or objects chosen from a larger group or population by a process giving equal chance of selection to all possible people or objects, and all possible subsets of the same size.

Range (of a set of data). The numerical difference between the largest and smallest values in a set of data.

Ratio table. A list of equivalent ratios organized by columns or rows.

Example:

Yellow Parts	Red Parts	Orange Sunglow Parts
3	4	7
12	16	28
[blank]	[blank]	[blank]
[blank]	[blank]	[blank]

Recursive pattern or sequence. A pattern or sequence wherein each successive term can be computed from some or all of the preceding terms by an algorithmic procedure.

Rekenrek. An arithmetic rack with two rows of 10 beads each, used as a tool for developing skill with counting, addition and subtraction.

Representation. An expression of a mathematical situation using pictures, words, numbers, tables, and/or equations.

Revoice. A teacher talk move in which the teacher restates or rephrases a student's mathematical statement in more formal and/or more precise terms.

Rigor. This framework interprets rigor to mean that conceptual understanding can be used to analyze a novel situation encountered in the world. Rigor means that students understand and can flexibly apply methods to different situations, connect mathematical ideas, approaches, and representations. The Drivers of Investigation provide reasons to think rigorously so that links through and among Content Connections are recognized, valued and internalized. Rigorous reasoning enables understanding “all the way down to the bottom” (Ellenberg, 2014, 48).

Right angle. A 90-degree angle.

Scalene. A type of triangle in which no two sides are equal in length.

Scaling. The process of multiplying each length in a diagram or figure, or parts of a ratio, by a fixed quantity, known as a scale factor, to enlarge or shrink, and preserve the relative sizes of all pieces.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.

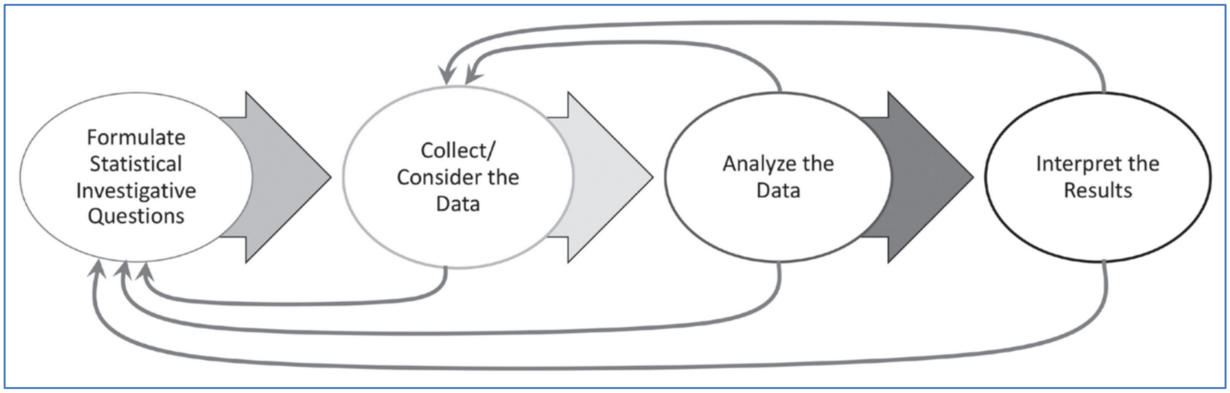
Sociocultural. Combining social and cultural factors.

Sociopolitical. Combining social and political factors.

Standards-based grading. See mastery-based grading.

Standard algorithm. A step-by-step approach to calculating, decided by societal convention, developed for efficiency. Flexible and fluent use of standard algorithms requires conceptual understanding.

Statistical and data science investigation process. A four-part process (graphic from Bargagliotti et al., 2020)



Strategy. Mental or written method chosen for approaching or solving a problem; may be invented by a student.

Strip Diagram. A rectangular visual model resembling a strip of paper or tape, with divisions used to assist mathematical calculations. Also known as a bar model, length model or tape diagram. It is used to solve word problems.

Subitize. To recognize a small quantity of objects without having to count them singly.

Symmetry. The quality of being made up of exactly similar parts facing each other or around an axis.

Tangent. (a) A line passing perpendicular to a radius at the point lying on the circle is said to be tangent to the circle. (b) The trigonometric function that, for each input of an angle, has an output that is the quotient of the y-coordinate divided by the x-coordinate for the point on the unit circle corresponding to the angle. (c) For an acute angle of a right triangle, it is the ratio between the leg opposite the angle and the leg adjacent to the angle.

Technology-rich environment. A setting in which the technology serves a clearly defined pedagogical purpose. This is distinguished from a techno-centrist educational approach, in which the use of technology is both a means and an end, where the primary goal is for students to learn how to use the technology.

Three Reads Strategy. A reading protocol for integrated ELD where students first read to understand, then read to identify and understand the mathematics, then read to make a plan. Their discussion is framed by cues for these stages on the board.

Trigonometric functions. The three common trigonometric functions are sine (sin), cosine (cos), and tangent (tan). Each involves the coordinates (x coordinate for cosine, y coordinate for sine, and quotient y divided by x for tangent) of points on a unit circle in the unit circle model of trigonometry. Or, the trigonometric functions can be considered as ratios involving the sides of a right triangle.

Trigonometry. The branch of mathematics involving the relationships between angles, points on the unit circle, and the sides/angles of triangles. These relationships are known as the trigonometric functions.

Two-way frequency table. A way to display the frequencies for two categorical variables.

Example: Two-way table showing favorite board games.

Age	Chess	Checkers	Monopoly	Total
Under 10 years of age	6	16	9	31
11–20 years of age	10	7	15	32
Over 20 years of age	12	8	12	32
Total	28	31	36	95

Unit fraction. A fraction with a numerator of 1.

Variable. A quantity that can change or that may take on different values. Refers to the letter or symbol representing such a quantity in an expression, equation, inequality, or matrix. (Source: Mathwords, 2013.)

Whole numbers. The numbers 0, 1, 2, 3, …

Tables

Grades TK–5

Table of Common Addition and Subtraction Situations (as found on p. 860 of 2013 *Mathematics Framework* Glossary)

Common Addition and Subtraction Situations*

Common Addition and Subtraction Situations	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ☐	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were 5 bunnies. How many bunnies hopped over to the first two? 2 + ☐ =	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were 5 bunnies. How many bunnies were on the grass before? ☐ + 3 =
Take from	Five apples were on the table. I ate 2 apples. How many apples are on the table now? 5 – 2 = ☐	Five apples were on the table. I ate some apples. Then there were 3 apples. How many apples did I eat? 5 - ☐ =	Some apples were on the table. I ate 2 apples. Then there were 3 apples. How many apples were on the table before? ☐ – 2 =

Common Addition and Subtraction Situations	Total Unknown	Addend Unknown	Both Addends Unknown†
Put together/Take apart‡	Three red apples and 2 green apples are on the table. How many apples are on the table? 3 + 2 = ☐	Five apples were on the table. Three are red, and the rest are green. How many apples are green? 3 + ☐ =, - 3 = ☐	Grandma has 5 flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

Common Addition and Subtraction Situations	**Difference Unknown**	**Bigger Unknown**	**Smaller Unknown**
Compare**	(“How many more?” version): Lucy has 2 apples. Julie has 5 apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has 2 apples. Julie has 5 apples. How many fewer apples does Lucy have than Julie? 2 + □ =, - 2 = □	(Version with *more*): Julie has 3 more apples than Lucy. Lucy has 2 apples. How many apples does Julie have? (Version with *fewer*): Lucy has 3 fewer apples than Julie. Lucy has 2 apples. How many apples does Julie have? 2 + 3 = □, 3 + 2 = □	(Version with *more*): Julie has 3 more apples than Lucy. Julie has 5 apples. How many apples does Lucy have? (Version with *fewer*): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? - 3 = □, □ + = 5

*Adapted from Boxes 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (National Research Council, Committee on Early Childhood Mathematics 2009, 32–33).

‡Either addend can be unknown, so there are three variations of these problem situations. “Both Addends Unknown” is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

†These take-apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign (=), help children understand that the equal sign does not always mean *makes* or *results in*, but does always mean *is the same number as*.

**For the “Bigger Unknown” or “Smaller Unknown” situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

Table of Common Multiplication and Division Situations (as found on p. 861 of 2013 *Mathematics Framework* Glossary)

Common Multiplication and Division Situations	Unknown Product	Group Size Unknown	Number of Groups Unknown
n/a	= □	□= and ÷= □	□= and ÷ = □

Common Multiplication and Division Situations	Unknown Product	Group Size Unknown	Number of Groups Unknown
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there altogether? Measurement example You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally and packed inside 3 bags, then how many plums will be in each bag? Measurement example You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed, with 6 plums to a bag, then how many bags are needed? Measurement example You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[†], Area[‡]	There are 3 rows of apples with 6 apples in each row. How many apples are there? Area example What is the area of a rectangle that measures 3 centimeters by 6 centimeters?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example A rectangle has an area of 18 square centimeters. If one side is 3 centimeters long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? Area example A rectangle has an area of 18 square centimeters. If one side is 6 centimeters long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example A rubber band is 6 centimeters long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18, and that is three times as much as a blue hat costs. How much does a blue hat cost? Measurement example A rubber band is stretched to be 18 centimeters long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? Measurement example A rubber band was 6 centimeters long at first. Now it is stretched to be 18 centimeters long. How many times as long is the rubber band now as it was at first?
General	= □	□= and ÷= □	□ $xb=p$ and $p \div b=$ □

*The first examples in each cell focus on discrete things. These examples are easier for students and should be given before the measurement examples.

† The language in the array examples shows the easiest form of array problems. A more difficult form of these problems uses the terms rows and columns, as in this

example: “The apples in the grocery window are in 3 rows and 6 columns. How many apples are there?” Both forms are valuable.

‡ Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps; thus array problems include these especially important measurement situations