Mathematics Framework
Chapter 10: Supporting Educators in Offering Equitable and Engaging Mathematics Instruction

Second Field Review Draft

Introduction

A broad system of support is needed to ensure that all students have access to mathematics instruction that reflects authentic contexts and real-world problems, is rich with connections between mathematical ideas and with students’ lives, and builds over time. As students learn and process mathematics, their teachers learn the effects of their practice and refine their teaching; together these processes form the core learning environment for mathematics. So, how can teachers be best supported in creating equitable and engaging mathematics learning environments for their students? Administrators and teacher leaders, such as coaches and teachers on special assignment, provide the initial, programmatic layers of support, while parents, counselors, and community members co-create an interconnected system that supports children and adolescents as they learn. This chapter presents guidance designed to build for teachers an effective system of support as they facilitate learning for their students.

Start callout box

Authentic: An authentic problem, activity, or context is one in which students investigate or struggle with situations or questions about which they actually wonder. Lesson design should be built to elicit that wondering. In contrast, an activity is inauthentic if students recognize it as a straightforward practice of recently-learned techniques or procedures, including the repackaging of standard exercises in forced “real-world” contexts. Mathematical patterns and puzzles can be more authentic than such “real-world” settings.

End callout box

It is crucial that anyone making professional learning plans for mathematics teachers understand the vision of mathematics teaching and learning described in this framework. As described in Chapter 2, the goal “is for students to view mathematics as a vibrant, inter-connected, beautiful, relevant, and creative set of ideas.” Chapter 2 develops five central themes of instruction that develop this view of mathematics.

1. Plan teaching around big ideas.
2. Use open, engaging tasks.
3. Teach toward justice.
4. Invite student questions and conjectures.
5. Center reasoning and justification.

In addition, Chapter 2 presents a framework drawn from Darling (2019) that is important for supporting linguistically and culturally diverse English learners, as well as other students:

1. Take an asset approach and recognize multilingualism as a power.
2. Include group work (strategically grouping for language development).
3. Make work visual (include graphic organizers, visual examples, encourage visual communication).
4. Build on students' lived experiences and cultures (allow native language use).
5. Scaffold learning and language development (sentence frames, sentence starters).
6. Give opportunities for pre-learning (giving students opportunities to learn some prerequisite material ahead of time).

Professional learning experiences for teachers, teacher leaders, and administrators must be designed to support instruction that implements these themes.

The framework’s progression chapters (Chapters 3–5) illustrate instruction through the development of major mathematical strands—mathematical practices and content—across the full transitional kindergarten through grade twelve grade continuum, and grade band chapters (Chapters 6–8) further detail ways educators can maintain a focus on big ideas and implement instruction in developmentally-appropriate ways. Classroom activities designed around big ideas will typically pair one or more Content Connections (CC; broad categories of mathematical content) and one or more Standards for Mathematical Practice (SMP) with a Driver of Investigation (DI; purposes for pursuing mathematical learning). Because instruction is so tied to these three dimensions, these three dimensions should also play a major role in the design of professional learning.

Drivers of Investigation

- DI.1: Making Sense of the World (Understand and Explain)
- DI.2: Predicting What Could Happen (Predict)
- DI.3: Impacting the Future (Affect)

Content Connections

- CC1: Communicating Stories with Data
- CC2: Exploring Changing Quantities

- CC3: Taking Wholes Apart, Putting Parts Together
- CC4: Discovering Shape and Space

Standards for Mathematical Practice

- SMP.1: Make sense of problems and persevere in solving them
- SMP.2: Reason abstractly and quantitatively
- SMP.3: Construct viable arguments and critique the reasoning of others
- SMP.4: Model with mathematics
- SMP.5: Use appropriate tools strategically
- SMP.6: Attend to precision
- SMP.7: Look for and make use of structure
- SMP.8: Look for and express regularity in repeated reasoning

In an attempt to provide some consistency across subjects for those seeking to create opportunities for professional learning, this chapter of the framework mirrors in structure Chapter 12 (Implementing High-Quality Science Instruction: Professional Learning, Leadership, and Supports) of the *California Science Framework* (CDE, 2016), and echoes many of its recommendations for supporting quality instruction.

Collaborative Systems of Learning and Support

Teachers perform incredibly complex work that relies on thousands of instructional decisions every day (Ball, 2018): in understanding their students' thinking, choosing tasks, deciding which questions to pose in discussion, selecting which (and whose) lines of inquiry to pursue with the class, and ensuring that all students have their authentic and culturally relevant contexts and tasks represented. When educational partners and influencers outside of the classroom are not aligned, for example when a textbook does not align with the vision of classroom instruction, this work of teaching is made even more difficult, and instructional practice changes little.

The California Common Core State Standards in Mathematics (CA CCSSM) were adopted by the State Board of Education in 2010. While the standards implementation has led to significant change, the iterative nature of teaching means that improvement is ongoing. The continuous improvement of mathematics teaching and learning requires the aligned efforts of many educational partners and communities (adapted from the *California Science Framework*, 2016):

- Teachers and teacher leaders prepared to engage in student-centered teaching that engages students in equity-oriented learning through authentic tasks and contexts that are relevant to those students based on their choices, interests, and aspirations
- School, district, and county office administrators who are knowledgeable and supportive of the changes demanded by the CA CCSSM and this framework
- Afterschool, early childhood, and other expanded learning opportunities aligned with and supportive of authentic mathematics learning that include collaborative and coherent efforts between teachers and other education support professionals
- College and university faculty involved in and advocating for high-quality mathematics instruction and preparation of future teachers
- Community members and parents, guardians, and families who understand the reasons for and are supportive of engaging and equitable approaches to mathematics teaching and learning
- Formal and informal learning environments, including museums, libraries, science centers and other venues that are fully committed to supporting the CA CCSSM

Effective progress takes place within these communities when it is aligned with an ongoing cycle of implementation, reflection, and improvement of practice (Little, 2006; Penuel, Harris, and Debarger, 2015; Fixsen, Naoom, Blase, Friedman, and Wallace, 2005; Fixsen and Blase, 2009). The vision is for teachers and other educational partners to engage in a learning community that has the same characteristics—respect, commitment, intellectual engagement, and motivation toward continuous improvement—that all educators hope to create for students in California classrooms.

Ermeling and Gallimore (2013) present models of implementation that have been embedded in school learning communities across 40 districts. These models focus on addressing learning needs common to the members of the community; analysis of evidence is used to drive planning, decision making, and critical questioning of practices. To be effective, the learning community must operate in an environment of collaboration and trust among teachers and school leaders, each of whom recognize that improvement requires time, resources, continuous support, and an appreciation of risk-taking as new instructional approaches are implemented.

An environment that realizes these improvement efforts in mathematics teaching and learning should focus on the sustainability of the instructional practices and education programs—and the sustainability of the professional learning cycle itself—by fostering a collaborative school culture that engages educators, administrators, students, parents, guardians, families, education professionals, and community members (Fixsen and Blase, 2009). Establishing culture allows all educational partners to understand themselves as advocates and supporters in the effort to improve students' experience and achievement in mathematics.

Finally, the 2014 *California English Language Arts/English Language Development Framework (ELA/ELD Framework)* calls on teachers and educational leaders to examine personal beliefs and attitudes toward students and their families; the call also certainly applies to teachers across contents, including mathematics instruction. Explicit reflection helps educators approach all students with a growth mindset disposition that both values the cultural resources and linguistic assets students bring to the mathematics classroom and supports them to use these resources while expanding and adding new perspectives and ways of appropriating and using mathematics. Put simply, teachers' beliefs about their students significantly affect those students' motivation, experience, and achievement (Stipek, Givvin, Salmon, and MacGyvers, 2001; Heyder, Weidinger, Cimpian, and Steinmayr, 2020).

As mathematics teaching and learning are complex endeavors (Russ, Sherin, and Sherin, 2016), the complexity of teaching will be a recurring theme throughout this chapter. Indeed, even defining what is meant by improvement of teaching practice involves connected changes in general pedagogy, mathematics pedagogical content knowledge (“ways of representing and formulating the subject that make it comprehensible to others” [Shulman, 1986]), and mathematical knowledge for teaching (“the mathematical knowledge needed to carry out the work of teaching mathematics” [Ball, Thames, and Phelps, 2008]).

Professional Learning for Equity and Engagement: Critical Content

Mathematics education has a long history of inequitable access to rich learning (see Chapters 1, 2, and 9 for more discussion of this topic). It is incumbent on all in education, at state, county, district, site and departmental levels, to work together in creating, adapting, and implementing professional learning experiences that are designed to help teachers challenge and overcome the legacy practices that continue to perpetuate these inequities in access and attainment. Even when professional learning is designed with a different primary focus (mathematical practices, particular instructional routines, or teaching from big ideas, for instance), the implementation of these ideas should reflect in culturally relevant and sustaining ways, such as reliance upon cultural backgrounds and other funds of knowledge, and include awareness of and attention to the impacts of unconscious bias on students’ experiences in the mathematics classroom.

More importantly, the field should prioritize professional learning opportunities that focus primarily on equity in mathematics education. Equity cannot be an afterthought to more traditional mathematics content-centered offerings that do nothing to address the fact that “Black, Latinx, Indigenous, women, and poor students, have experienced long histories of underrepresentation in mathematics and mathematics-related domains” (Martin, 2019; see also Martin, Anderson, and Shah, 2017). Inequities caused by systemic issues means that a “culture of exclusion” persists *even in equity-oriented teaching* (Louie, 2017). Many of the stories used to define mathematics, and to talk about who does or is good at mathematics, are highly racialized and English language-centric, and are experienced that way by students (Lue and Turner, 2020). This means students’ mathematics identities are shaped by social messages that are conditioned by assumptions about race and gender. Professional learning in mathematics can respond to these realities and aim for more than incremental change (which does little to change the framing narratives that drive inequities).

It is important for educators to be provided with explicit connections, references and links to descriptions and supports for the implementation **of English Learner centered strategies** such *as sentence frames, leveled prompts, vocabulary banks, cognate study, intentional groupings, and the use of primary language as support,* among others. These will provide purposeful experiences for English learners to engage with language and mathematical concept development, as they deepen their knowledge of the Standards for Mathematical Practices. In addition to the resources listed below, several vignettes in this Framework, especially in Chapters 2, 6 and 7, include specific guidance to help teachers understand and implement instruction that supports English learners. In addition, in the transition to increased hybrid and in-person learning, accommodations and connections to ELD standards and online resources should be explicitly addressed, applied and incorporated into in-person, virtual asynchronous and synchronous lessons, as well as making explicit connections to the continued use of online multilingual resources and the use of on-line platforms and communication of expectations for both students and parents.

The table below (adapted from the 2014 *ELA/ELD Framework*) outlines critical content for professional learning. Due to the inherent complexity of teaching, there is a risk of trying to do everything at once; it is important to design opportunities around a manageable subset of these foci.

Critical Content for Professional Learning in Mathematics Education

Establishing a Vision for California's Students

- Develop the readiness for college, careers, and civic life
- Attain the capacities of numerate individuals
- Become broadly literate in quantitative subjects
- Acquire the skills for living and learning in the twenty-first century

Understanding the Standards

- CA CCSSM Mathematical Practice Standards
- CA CCSSM Mathematics Content Standards
- ELA/ELD Standards as implemented in Mathematics Classes
- Implementing science, history/social studies, career and technical education, and other standards in tandem with mathematics

Establishing the Context for Learning

- Integrating the curricula
- Motivating and engaging learners
- Teaching from Big Ideas, not individual standards
- Respecting learners, and the cultural and linguistic assets they bring
- Ensuring intellectual challenge

Enacting the Key Themes of Mathematics Instruction

- Mathematics as tools for solving authentic problems in authentic contexts
- Meaning making
- Mathematical practices
- Language development
- Effective expression
- Content knowledge

Addressing the Needs of Diverse Learners

- Comprehensive English language development: integrated and designated ELD
- Additive approaches to language and mathematics development
- Meeting the needs of students with disabilities and students experiencing difficulty
- Meeting the needs of advanced learners and other populations

Exploring Approaches to Teaching and Learning

- Teaching through investigation
- Models of instruction
- Culturally and linguistically responsive teaching
- Supporting biliteracy and multilingualism

- Supporting students strategically (including Universal Design for Learning [UDL] and the Multi-Tiered System of Support [MTSS])

Sharing the Responsibility

- Collaborating within and across grades, departments, and disciplines
- Promoting teacher leadership
- Partnering with community groups and higher education
- Collaborating with parents

Evaluating Teaching and Learning

- Types and methods of assessment (formative, summative, rubrics, portfolios, diagnostic)
- Cycles of assessment (short, medium, long)
- Student involvement in assessment
- Appropriate preparation for state assessments

Integrating Twenty-First Century Learning

- Critical thinking skills
- Creativity and innovation skills
- Communication and collaboration skills
- Community awareness leading to global awareness and competence
- Technology skills

Professional Learning Throughout a Teacher’s Career

Teachers learn to improve their practice in many contexts: working with students in the classroom, interacting with peers, communications from administrators, attending conferences, enrolling in online courses, and reading publications, to name a few. In this framework, *professional learning* refers to planned and organized processes that actively engage educators in cycles of continuous improvement guided by the use of data and active inquiry around authentic problems and instructional practices (Coggshall 2012). Darling-Hammond, Hyler, and Gardner (2017, v) use the related phrase, *effective professional development*, to mean structured professional learning that results in changes in teacher practices, which is vital to improving student learning outcomes. This section describes important aspects of professional learning at different stages of an educator's career, with particular focus on characteristics of effective professional learning. This is followed by considerations for planning effective professional learning. The section concludes with discussions of various models and strategies for professional learning, with several vignettes illustrating the models and their incorporation of the characteristics of effective professional learning.

Figure 10.1, adapted from the National Comprehensive Center for Teacher Quality's publication *Toward the Effective Teaching of New College- and Career-Ready Standards: Making Professional Learning Systemic* (Coggshall, 2012), summarizes key shifts in thinking about professional learning that will help improve teaching practice.

Figure 10.1

MOVING FROM	MOVING TOWARD
Believing that professional development is some people's responsibility	Believing that professional learning focused on student learning outcomes is everyone's job
Thinking individual goals for professional development are separate from school site and district goals	Aligning individual goals with school site and district goals to provide greater coherence
Using professional development as a means of addressing deficiencies	Embedding professional learning in continuous improvement
Seldom addressing standards for professional learning	Using standards for professional learning
Providing professional development that takes place outside of school, away from students, and is loosely connected to classroom practice	Embedding professional learning in the daily work of teaching so that staff can learn collaboratively and can support one another as they address real problems and instructional practices of their classrooms
Engaging staff in professional development unrelated to data and the continuous improvement process	Engaging staff in a cycle of continuous improvement, guided by the use of active inquiry and multiple sources of evidence

MOVING FROM	MOVING TOWARD
Providing one-shot or short–term professional development with little or no transfer to the classroom	Sustaining continuous professional learning through follow-up, feedback, and reflection to support implementation in the classroom
Limiting professional development based on scarce resources and discrete funding sources	Dedicating and reallocating resources to support professional learning as an essential investment

Source*:* Coggshall, 2012.

Teacher Preparation

Since CA CCSSM-aligned instruction is different in significant ways from the school mathematics experience of most teachers, the phases of new teacher preparation and induction are key factors in providing a pipeline of teachers with the skills and knowledge to provide high-quality CA CCSSM-aligned instruction. Educators of pre-service teachers need to align their programs to reflect the authentic-context, big-idea based instruction described in this framework so that pre-service teachers have the opportunity to experience it as learners. Factors to consider in the development of CA CCSSM-aligned teacher preparation programs include the following:

- Early field experience hours that are dedicated to observing and interacting with students and teachers in authentic mathematics classroom environments
- Student teaching opportunities that include content-rich experiences and integrated learning experiences
- Mathematics and mathematics methods classes that address mathematics as a collection of tools and lenses for making sense of authentic contexts, with emphasis on learning mathematical ideas through the mathematical practices and active-learning pedagogy rather than passive lecture
- Mathematics and mathematics methods classes that develop mathematics through asset-based, culturally- and linguistically-relevant and sustaining pedagogy

- Mathematics methods classes that address pedagogical content knowledge that facilitates student conceptual understanding of content standards over time and how to address incorrect, developing, and alternative student conceptions of those ideas
- Student teaching experiences with mathematics teachers who are effectively incorporating CA CCSSM
- Effective examples of the development of mathematical ideas through the investigation of authentic contexts and problems (in both pre-service teacher course work and student teaching)
- Mathematics methods classes that address how to organize instruction around big ideas and meaningful investigations, rather than isolated standards
- Mathematics and mathematics methods classes that explore mathematics, and the teaching and learning of mathematics, from many cultures. By taking the time to acknowledge and center contributions to mathematical understanding from Africa, South America, Asia, and indigenous peoples around the world, students can better appreciate the global nature of mathematical discovery. In a similar way, prospective teachers in methods courses can expand their understanding of teaching and learning mathematics by exploring a variety of approaches from a diverse array of cultures. Mathematics methods classes can make evident ways in which language and content are interconnected and mutually reinforcing: one cannot develop without the other. Language needed for disciplinary thinking and concepts should not be taught in isolation, but in the context of what students relate to and need to know to access and communicate mathematical thinking. Opportunities to practice language and communicate understanding must be integrated (e.g., students have the opportunity to gain ideas from a discussion or a reading before writing).

Additionally, mathematics education faculty and other educators (e.g., university field advisors, master cooperating teachers) who provide pre-service instruction must be grounded in the knowledge and skills within the context of CA CCSSM to facilitate their students' (pre-service teachers) ability to address the vision of the CA CCSSM. Other publications are also important resources for guiding the design of high-quality teacher preparation programs, including the Learning Policy Institute's *Effective Teacher Professional Development* (Darling-Hammond, Hyler, and Gardner, 2017), *Preparing Teachers—Building Evidence for Sound Policy* (NRC, 2010), *Powerful Teacher Education, Lessons from Exemplary Programs* (Darling-Hammond, 2006), and NCTM's Professional Development Guides (NCTM, n.d.).

Induction for New Teachers

Teaching is hard and thoughtful work. It is not uncommon for new teachers to feel isolation and burdened by the demands (both managerial and instructional) of preparing for and working in a classroom. Yet, the implementation of effective preparation and support programs specifically tailored to the needs of new teachers can alleviate these issues to a large degree. The following considerations can provide support for prospective teachers of mathematics:

- Redefine the professional dynamics of the teacher induction process by pairing beginning mathematics teachers with experienced mathematics teachers who can act as mentors rather than delegators. This connection may help address the need for inclusion and community, and may provide the new teacher a sense of ownership of the content and a sense of belonging in the mathematics department, leading to greater teacher retention.
- Recognize and support the need for elementary teachers to receive math-specific support and mentoring (see Content Focused section following).
- Ensure that beginning mathematics teachers have comparable access to mathematics teaching resources (including technology, teaching spaces, and materials for hands-on instruction) as other mathematics teachers in the school.
- Involve new teachers in available Professional Learning Communities, Lesson Study, or the like, particularly math-specific ones, in order to promote and aid regular reflection on their practice (Fulton and Britton, 2010).
- Encourage new teachers to attend mathematics teacher conferences, institutes, and workshops (and financially support them to do so).
- Ensure that beginning teachers understand who their students and families are, in particular their emerging multicultural learners, their interests, aspirations, and

cultural and environmental backgrounds and how to use those as resources for learning.

Ongoing Professional Learning for In-service Teachers

Characteristics of Effective Professional Development

Though there are many approaches to professional development—along with multiple aspects to each approach—some strategies and components have been shown to be more effective than others. NCTM in *Principles to Action* (2014) connects education research to teaching practice with professional learning materials to help educators learn specific, research-based teaching practices. The Learning Policy Institute's review of 35 rigorous studies on the implementation of professional development for teachers noted several elements of effective professional development that ultimately improve student outcomes (Darling-Hammond, Hyler, and Gardner, 2017). These elements, described below, include a focus on the following:

- content
- active learning
- collaboration
- modeling
- coaching
- feedback and reflection
- sustained engagement

Content Focused

Professional development in any discipline has been found to be most effective when the content knowledge in that area—in this case mathematics—is a primary focus. Teachers must have opportunities to explore mathematical big ideas through rich, authentic, culturally-relevant tasks in order to both deepen their own understanding of mathematics and better anticipate the challenges students might encounter and the strategies they may rely on to respond to them. These big ideas include the mathematical practices as central aspects of mathematics, equal in import to content standards. Professional development that introduces perspectives or teaching approaches without intentional connections to mathematics is unlikely to bring about much change in teachers' practice. Professional development that blends pedagogical and learning knowledge with mathematics knowledge has much more potential to result in powerful changes in students' learning experiences than that which focuses on pedagogy or content knowledge separately.

Many teachers have experienced mathematics as a set of procedures to be memorized. This narrow understanding makes access to opportunities to experience mathematics differently themselves all the more important, lest their own students have their mathematics identities shaped by similarly limited experiences of mathematics. As described in Chapter 1, the goal is that students achieve conceptual understanding, problem solving capacity and procedural fluency (in the full sense of the word fluency introduced in Chapter 1) in mathematics. When teachers work on rich, authentic, culturally-relevant mathematics tasks—through which they can ask their own questions, reason and communicate with others, develop curiosity and wonder—they start to see mathematical connections that they may never have seen before. This often changes teachers' relationships with mathematics, which is an important precursor to changing their teaching (see also Anderson, Boaler, and Dieckmann, 2018). This experience takes time and needs to be carefully organized, with teachers working together on mathematics in a supportive environment with an expert facilitator. Face-to-face professional development is the ideal way to encourage this experience, but online courses can also provide this experience, especially when teachers receive funded time to take the courses in groups.

Based in Active Learning

Teachers benefit most from professional development that engages them in the process of actively designing and trying teaching strategies, and provides them with opportunities to engage in the same style of learning they are designing for their students. Such professional practice relies on authentic artifacts, interactive activities, and other strategies to provide deeply embedded, highly contextualized professional learning. This approach moves away from traditional learning models and environments that are lecture based and fail to connect to teachers’ classrooms and students. Instead, teachers should have opportunities to make sense of student thinking (in order to assess students’ funds of knowledge and other assets—such as reasoning and communication practices—that will help drive teacher actions), reflect on their own and one another’s instructional practices, and discuss connections to their own classroom. Classroom video is a powerful resource for such reflections and discussions. For example, professional development may include opportunities to watch videos showing linguistically and culturally diverse communities of English learners working to high levels with an expert teacher. Videos and other records of practice such as student work, should be at the center of professional development opportunities.

Includes Collaboration

Effective professional development requires time and resources for teachers to share ideas and collaborate in their learning, often at the school level. Working collaboratively allows teachers to create professional learning communities that can positively change the culture and instruction at a classroom, grade, department, school, or district level. As teachers work together on mathematics instruction, they experience the collaborative, connected mathematics experience as a template for their own classrooms. They can also share experiences, including challenges, successes, and insights, to support one another in planning and implementing lessons. Professional learning communities are also important places to consider ways in which mathematics instruction can recognize students’ cultural and linguistic assets, to make contexts and problems ever-more authentic for students.

Uses Instructional Examples

Seeing lessons, tasks, and curriculum in action is a powerful tool for providing teachers with opportunities to see best practices first hand. Teachers may view examples that include lesson plans, unit plans, sample student work, observations of peer teachers, and video or written cases of teaching, such as the many vignettes and snapshots presented in this framework. Teachers benefit from opportunities to discuss examples of teaching, reflect on current practices, and make connections to their own classrooms.

Effective professional learning must build teachers’ capacities to notice, analyze, and respond to students’ thinking (NCTM, 2014, 101), and professional learning built around artifacts of practice such as student work (written, video, or other) provides time and support to develop these capacities.

Provides Coaching and Expert Support

Implementing new teaching approaches can shift particular classrooms, schools, or even districts. Fortunately, coaching and expert support—especially from district and county mathematics coaches—has proven extremely effective to respond to these changes when it is structured around a particular purpose (for example, adopting new curriculum or implementing specific new instructional practices) and is aligned with school-wide goals and priorities. Well-trained peers and teacher leaders with expertise in particular approaches can be powerful facilitators of growth in encouraging, modeling, and sharing insight—particularly when supported by administration and appropriate structure. These leaders can spend time observing teachers’ instructional practices, recognize assets that teachers can build on, and work with teachers toward ever-growing capacity to implement rich, student-centered mathematics lessons.

Includes Feedback and Reflection

High-quality professional development ensures teachers are afforded dedicated time to think about, receive input on, and make changes to their practice. They can facilitate reflection and solicit feedback, both of which enable teachers to establish and refine realistic goals of changing practice as they move toward expert visions of practice.

Has a Sustained Duration

Effective professional development provides teachers with adequate time to learn, practice, implement, and reflect upon new strategies that facilitate growth in their practice. Professional development which engages teachers in making incremental changes over time (and reinforces existing practices) can bring about lasting positive changes.

Planning for Effective Professional Learning

Achieving this framework's vision of mathematics education will require improved systems of professional learning. Teachers, specialists, paraprofessionals, and school and district leaders should identify personal and collaborative learning goals that articulate across grade levels and departments, focusing on curriculum, instruction, and assessment strategies that embrace the vision of the CA CCSSM and this framework. The schools, districts, and other local education agencies (LEAs) must become "learning organizations" (Senge, 1990) that are engaged in continuous improvement around the teaching and learning of mathematics. At every level (grade, department, school, district) educators must share a vision that focuses on student learning, collaboration, collective inquiry, shared practices, reflection, and results (Louis, Kruse, and Marks, 1996; DuFour, 2004; Hord and Sommers, 2008). As discussed in the Role of Parents, Guardians, and Families section following, this shared vision includes collaborating with families, as educators and administrators can gain a better understanding of students' learning needs by considering them holistically.

County offices of education, districts, schools, and other LEAs providing professional learning can use the report Effective Teacher Professional Development (Darling-Hammond, Hyler, and Gardner, 2017) as a resource for planning these types of learning experiences. This report gives much more detail about the features of effective professional learning described above.

Another resource for those designing professional learning opportunities is *Professional Development Design Framework* (Loucks-Horsley et al., 2010). Through their research with national professional developers, Loucks-Horsley and her colleagues found that effective programs had several common characteristics. They were designed to meet various factors, to change over time, and to adapt to particular goals and contexts. They did not rely on formulas; instead, the designers used a process of thoughtful, conscious decision making. The authors used these factors and processes to create the framework as seen in figure 10.2 below.

Figure 10.2. Professional Development Design Framework

Source: Loucks-Horsley et al., 2010.

At the center of the design framework, illustrated in the six squares connected with horizontal arrows, is a planning sequence that includes the following topics: (1) committing to a vision and a set of standards; (2) analyzing student learning and other data; (3) goal setting; (4) planning; (5) doing; and (6) evaluating. The circles above and below the planning sequence represent important inputs into the design process that can help designers of professional learning make informed decisions. These inputs prompt designers to consider the extensive knowledge bases that can inform their work (knowledge and beliefs), to understand the unique features of their context, to draw on a wide repertoire of professional development strategies, and to wrestle with critical issues that instructional reformers will encounter.

While there is no exact starting place for using the design illustrated in Figure 10.2, effective planning should avoid starting with strategies—though they may seem most appealing. Instead, the use of evidence (derived through questions such as, What are the assets? or, What are the needs?) is encouraged. Additional considerations should be made, such as thinking about short- and long-term approaches (up to five years), considering teacher career trajectories, and supporting teachers accordingly (Task Force on Educator Excellence, 2012).

However, those developing professional learning must also remain mindful of the need to stay flexible and adaptive, and they should include openness to refining their ideas as they evaluate the implementation process. As the design and implementation phases are taking place, recommendations from Innovate: A Blueprint for Science, Technology, Engineering, and Mathematics in California Public Education (STEM Task Force, 2014) and the characteristics of effective professional learning should also be considered during the design phase.

For consideration: while the Professional Development Design Framework in Figure 10.2 is arranged as a linear and sequential model, it need not be employed as such. What is most important is to pay attention to the four core design inputs, where they impact the design of the program, and how they are addressed during implementation.

Models and Strategies: Effective Professional Learning

The characteristics of effective professional learning can be implemented through many professional development models and strategies, including the following:

Models

- Professional Learning Communities (PLCs): opportunities for teachers to collaborate with each other, and for administrators to collaborate with their teachers, in a team setting
- Communities of Practice are "...groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly" (Wenger-Trayner and Wenger-Trayner, 2015). In educational settings, PLCs are often site-based, and Communities of Practice often connect educators across sites, helping provide additional contacts and resources for improving practice.
- Classroom coaching: A **mathematics coach** is an individual who is well-versed in mathematics content and pedagogy and who works directly with classroom

teachers to improve student learning of mathematics (Hull, Balka, and Miles, 2009).

- Lesson Study (see below)
- Mathematics Labs: Collaborative design and instruction cycle, similar to Lesson Study but with collaborative instructional decisions even during the lesson's implementation (Kazemi et al., 2018)
- Content-intensive institutes with follow-up workshops (see below)

Strategies

- **Backward design**: importance of student learning outcomes in lesson design
- Implementation of and alignment with the guidelines of Universal Design for Learning (UDL)
- Networking and community building around mathematics instruction.
- Partnerships with university mathematics and mathematics education faculty: Bridging the research–practice divide

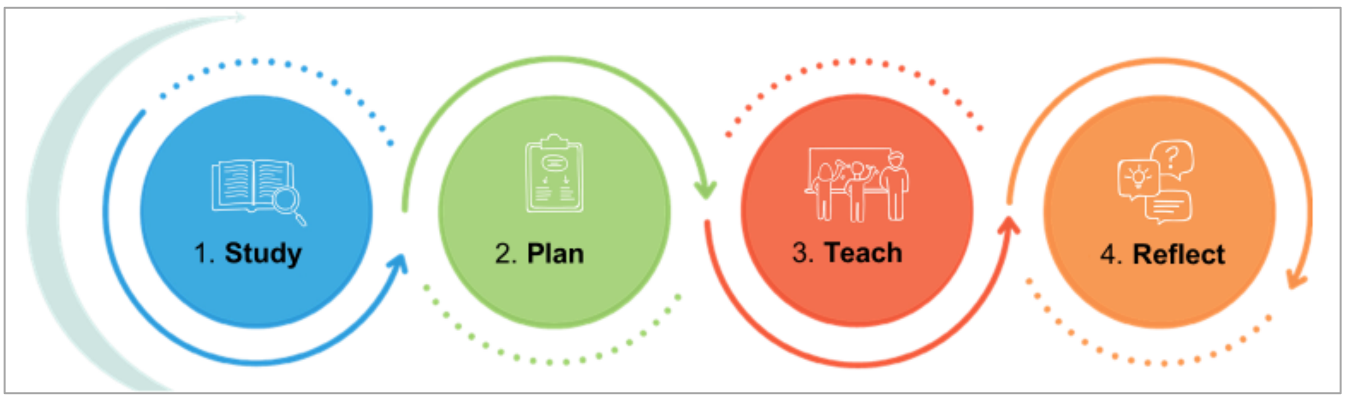
Three models that are supported by research into effective professional development in mathematics are explored below. The first, Lesson Study, offers sustained content-focused courses with school-year follow-up, and coaching. In a survey of the effectiveness of 643 professional development models, only two models were found to have a significant positive effect on students' learning—lesson study and sustained content-focused summer courses with pedagogy-oriented structured academic year follow-up (Gersten et al., 2014). Coaching models are very common in California schools, but "...there is little empirical evidence that coaching improves teacher practice" (Desimone and Pak, 2017). However, some structured coaching models show more promise for instructional improvement than individual one-on-one models (Gibbons, 2017).

Lesson Study

Lesson study is a type of professional learning where teachers engage in an inquiry cycle that supports their ability to experiment, observe and improve their teaching by collaboratively researching, creating, teaching/observing, and revising a lesson. Lesson study, which originated in Japan, has been shown to be an effective model for professional development with its deliberate focus on planning and teaching practice as well as inquiry, creativity, and collaboration (Lewis and Hurd, 2011).

The proven effectiveness on student learning led the California Mathematics Project (CMP) to formally adopt lesson study as a preferred means of professional development in 2018, and eventually spearhead the creation of the California Action Network for Mathematics Excellence and Equity (CANMEE, n.d.). CANMEE supports California schools and districts in their implementation of high-quality lesson study. The Lesson Study Group at Mills College provides many online resources to support such implementation.

The lesson study cycle consists of four phases (Mills College. n.d.)



In the Study phase, a team of teachers collaborates to:

- Identify long-term goals for students
- Choose the subject and unit to investigate
- Study standards, research, and curricula

In the Plan phase, using insights from the Study phase, the team:

- Examines the unit and chooses one lesson to plan in depth
- Articulates the lesson goals
- Tries the lesson task and anticipates student thinking

- Identifies data to be collected during the lesson

In the Teach phase, the team puts that lesson into action:

- One team member teaches the lesson
- Other team members observe and record student thinking and learning

In the Reflect phase, the team then reflects on their work by:

- Meeting after the lesson to discuss data on student thinking and learning
- Having an outside specialist provide further commentary
- Reflecting on what they learned during the cycle as a whole

Some or all of these phases are often repeated by a team as a team often wishes to redesign a lesson based on realizations made in the Reflect phase, and teach it again to another class of students.

It is important to note that the "product" of a lesson study cycle is more than a refined lesson plan: Team members deepen their understanding of content and student thinking, their commitment to collaboration, and their ability and inclination to base instructional decisions on evidence of their students' thinking.

Lesson Study Vignette

Grade level: Second

Equity focus: Linguistically and culturally diverse English learners' productive language use in mathematics

Source: The California Action Network for Mathematics Excellence and Equity (CANMEE) Steering Committee, adapted

The second-grade teachers at 54th Street Elementary met during their Professional Learning Community time to discuss the performance of their emerging multicultural learners in mathematics. Each teacher noticed that their English learners were having difficulty explaining their solutions to mathematics problems orally and in writing. They invited the English language development (ELD) specialist to the meeting to hear their concerns and obtain suggestions for addressing the students' needs.

The ELD specialist had recently observed a lesson at another elementary school focused on equity. The ELD specialist suggested that the second-grade teachers consider participating in a lesson study focused on building the agency of their multilingual students. The teachers decided to engage in a lesson study cycle of 30 hours and followed the lesson study model of study, plan, do/test, and reflect.

As part of the equity focus of the CANMEE lesson study process, each teacher selected four designated English learners as focal students from their classes and interviewed them to determine their strengths and challenges in mathematics. Based on the content of interviews and classroom observations, the teachers drafted assets-based descriptions for each, then met and shared their focal student descriptions.

During the Study phase of their lesson study, the teachers read literature that centered on effective practices for English learners, such as the *English Language Arts/English Language Development Framework* (CDE, 2014), the *English Learner Roadmap* (CDE, 2017), and important research (Moschkovich, 2012; Ramirez and Celedón-Pattichis 2012). As part of the Plan phase, teachers designed a mathematics lesson with a task that required students to record their thinking in a journal, and share their ideas with a partner. One of the goals for the focal students was to increase their productive language skills. The teachers engaged in the mathematics task themselves to anticipate both productive and unproductive student strategies. The teachers developed questions to ask those students who used unproductive strategies, and consulted with the ELD specialist for additional resources. The specialist posed questions to allow the teachers to do the thinking.

In the do/test phase, one of the teachers on the team volunteered to teach the lesson while the other teachers observed the lesson to determine the effect of the lesson they designed. An outside expert in mathematics content was invited to provide feedback on the mathematics content of the lesson, serving as the mathematics commentator. The ELD specialist served in the role of equity commentator. The ELD specialist observed the focal students' interaction with the lesson and peers, and their productive language skills—in particular, aspects of the lesson design that seemed to facilitate productive language opportunities. The second-grade teachers also invited other educational partners, including colleagues at the school and parents, to observe the public lesson.

After the lesson was taught, as part of the Reflect stage, the team of teachers shared their thoughts and observations about the implementation of the lesson, and identified ways to improve practice moving forward. The mathematics and equity commentators shared their observations of the lesson and provided suggestions for next steps. Other observers (including parents) also made comments about the lesson.

At the end of the cycle, the second-grade teachers reflected on the professional learning experience. They noted the value in the ability to collaborate with their peers about a problem of practice that was specific to their school. The teachers also felt that the support from the ELD specialist was critical to their success. They all noticed an increase in agency among the focal students as a result of the lesson study process. Lastly, the second-grade teachers noted feeling more confident about their ability to meet the needs of their students who are emerging multicultural learners.

Content-focused workshops with follow up

"One and done" professional development sessions have shown little impact on teaching practice or student learning (Darling-Hammond, Hyler, and Gardner, 2017). In addition to lesson study, sustained content-focused professional courses/workshops with school-year pedagogy-focused follow up have also demonstrated positive impact on student learning (Gersten et al., 2014). There are several partner organizations in California that work with districts and schools to provide these opportunities.

Professional Learning Vignette: Tulare County–Youcubed partnership

This vignette describes a model of professional learning which combines a focus on mathematical mindset and content knowledge, through a model of paid time where teachers can learn and plan together with shared goals and resources.

The Tulare County Office of Education partnered with Youcubed in offering a blended model of professional learning for teachers and leaders across 11 school districts. The partnership was called the Central Valley Networked Improvement Community (CVNIC). County leaders chose fifth grade as the focus of the work, as very low percentages of students in fifth grade either met or exceeded proficiency on the California Assessment of Student Performance and Progress (CAASPP). The table below shows the percentages of students involved in the initiative by ethnicity, socio-economic status, language learning and their proficiency results on the fifth-grade CAASPP tests:

Regional Schools	Student N	Percent Latino	Percent English Learner	Percent Low SES	Percent Proficient (All Students) on CA Grade 5, Test* (2016)
School 1	572	72%	28%	83%	8%
School 2	410	68%	35%	86%	17%
School 3	712	98%	64%	97%	7%
School 4	624	95%	63%	96%	8%
School 5	445	28%	42%	21%	5%
School 6	487	19%	68%	19%	3%
School 7	687	11%	58%	11%	4%

During the year-long partnership, teachers and their administrators were provided paid time to complete an online course. Upon completion of the course, teachers met in groups to discuss learning and plan classroom changes. The meeting time was facilitated by county office leaders who led full-day sessions centered on mathematics collaboration. The network focused on implementing structures that reinforced the importance of growth mindsets in mathematics and ways for students to see mathematics as a connected, visual subject, with classroom strategies that fostered this approach.

Many teachers shared that particular students, especially those designated as multilingual learners, had developed the idea they did not “have a math brain” and that mathematics was a set of procedures to memorize. This factored in their achievement levels.

The professional development sessions conducted by the county included engaging the teachers with rich mathematics tasks that were visual and showed the connected nature of mathematics (Youcubed, n.d.b). The teachers' work was informed by the research promoting the importance of struggle for brain development, and they were reminded that students remained capable of learning anything. The teachers in the networked community agreed to begin the school year with the "week of inspirational math" lessons (Youcubed, n.d.c). The schools conducted diagnostic surveys to learn about the students and their ideas about mathematics. These surveys were conducted at the beginning of the school year, and repeated again at the end of the year of the intervention.

Each time the teachers took a lesson from the online course, they met to discuss the changes the lesson would inform in their classrooms. As the year progressed the teachers continued to include the use of rich, visual, creative mathematics tasks with increasing frequency; this altered their textbook tasks, inspiring more flexible uses, and allowing them to rely more on students' ideas, how they use them, and share how they strategize in mathematics. The teachers reflected that this gave students—and the teachers too—new access to understanding. As one teacher shared:

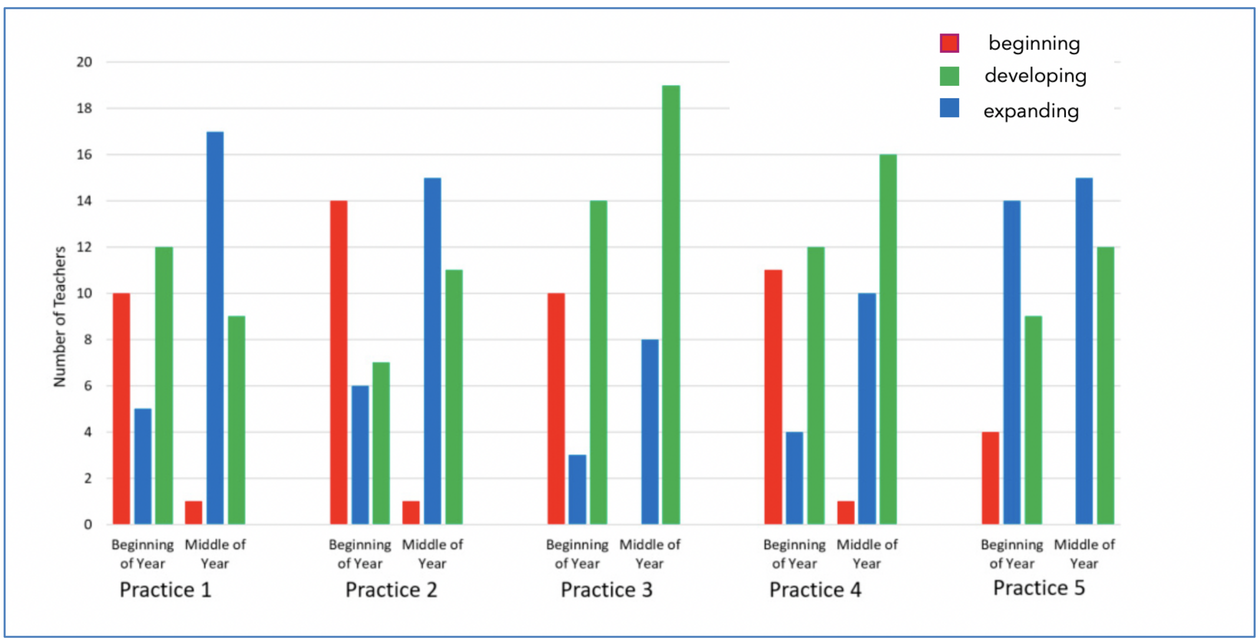
> When I first started this journey, I was always doing the algorithm because that was my safety net. Now I'm thinking, "Okay, how am I going to draw this? How do I visually see this?" Now I understand why the algorithm works, because I now have this totally clear picture in my head. Which has been a really good thing when it comes to things like fractions. And for these kids, it's like, "Oh, that's why it works."

Another teacher noted:

> Oh, the visuals.... They love that too, cause with their ideas of how it would form, and how they would build. I do it periodically. Just throw up a visual with different things and say, "Okay, what do you see? What don't you see? What might you see? What could be the next thing?"

The county leaders and Stanford team developed a mindset guide to help teachers and leaders understand the important aspects of a mathematical mindset (Youcubed, n.d.d). The guide includes advice for giving mindset messages, using rich tasks and emphasizing mathematical and student connections, and assessing students in ways that are compatible with a mindset approach. County officials observed classrooms at different intervals and recorded the teachers' practices in relation to the features of the guide. Figure 10.3 shows that the teachers developed their practice in relation to all five features of the guide. At the beginning of the year, high proportions of teachers were at the "beginning" level of the five classroom features, by the middle of the year, more teachers were at the "developing" and "expanding" levels, with three of the features reaching significance levels. For example, the teachers' practice on the "nature of mathematics" significantly improved (t = 3.03, p = 0.005).

Figure 10.3



Link to long description

The Mindset Guide, used for training teachers and as an observational tool:

Mathematical Mindset Practice 1: Growth Mindset Culture

Mindset	Beginning	Developing	Expanding
Mindset Messages	Brain and belief messages are never given or only to some students	Belief messages are given occasionally or too generically	Brain and belief messages are given in a meaningful way: “I know you can do this,” “As you learn this pathway forms in your brain”
Praising the Learning Process	Praise is focused on answers rather than effort and progress in thinking	Praise is sometimes focused on effort and process	Effort, ideas, and strategies are consistently recognized and praised
Students’ Mindsets	Students talk about some people being “math people” and some not	Students convey a mix of confidence and doubt in themselves	Students show self-belief and confidence

771 Mathematical Mindset Practice 2: Nature of Mathematics

Mindset	Beginning	Developing	Expanding
Open Tasks	Tasks are relatively closed, emphasizing procedures with little reasoning	Rich tasks are occasionally used	Tasks are mathematically rich in reasoning opportunities, allowing for different approaches and visuals
Reasoning and Multiple Perspectives	Maths work does not include reasoning, visuals, or multiple perspectives	Occasionally multiple methods and visuals are elicited and explored	Students use and share different ideas, visuals, and methods and use ownership words (e.g., “my method”)
Depth Over Speed	Strong emphasis on speed, memorization, and correct answers	Occasional emphasis on speed, memorization, and correct answers	Emphasis is on depth, creativity, visuals, and mathematical beauty

772 Mathematical Mindset Practice 3: Challenge and Struggle

Mindset	Beginning	Developing	Expanding
Mistakes	Complete and correct work is emphasized, mistakes are discouraged	Mistakes are acceptable but not explored	Mistakes are valued, students are comfortable sharing even if unsure

Mindset	Beginning	Developing	Expanding
Struggle & Persistence	Students expect and rely on teacher assistance when they struggle	At times struggle is celebrated, at others students are led to a solution	Struggle is valued; e.g., “this is the best time for brain growth.” Students persist longer
Questioning	Questions are low-challenge or narrowly focused	Deep-thinking questions are occasionally used	Questions are open and encourage multiple methods, ways of seeing, and thinking

Mathematical Mindset Practice 4: Connections and Collaborations

Mindset	Beginning	Developing	Expanding
Mathematical Connections	Maths is presented as a set of disconnected ideas	Connections are implied but seldom discussed	Connections between ideas, methods, and representations are highlighted and explored through visuals, movement, and creativity
Connecting in Small Groups	Student discussion is not encouraged	Student discussion is encouraged but only some students take part	Students collaborated and build off each other’s ideas and all students are involved
Connecting as a Whole Class	No opportunities for whole-class discussion	Class discussion is encouraged, e.g., “Does anyone want to respond to [blank]’s idea?” but most interactions are teacher-student.	Students talk directly to each other; the teacher is just one member of the mathematical community

Mathematical Mindset Practice 5: Assessment

Mindset	Beginning	Developing	Expanding
Nature of Feedback	Harsh grading on a curve, ranking, no revisions, punitive	Standards-based grading – with feedback on standards met. Revisions are not allowed	Assessment is used formatively, e.g., verbal, written, and ongoing feedback on specific learning goals. Revisions are encouraged
Frequency of Testing and Grading	Grades and tests/quizzes are frequent; performance culture	Grades and other summative measures are only given at the end of the unit	Learning culture with diagnostic feedback

Mindset	Beginning	Developing	Expanding
Multiple Forms of Assessment	Assessment is based on tests, quizzes, and homework. Focus is on answers only	Assessment includes more multidimensional evidence of learning, not only answers	Formative assessment valuing a broad form of mathematics – e.g., example visuals, making sense, multiple methods

The blended model of professional learning led to several changes over the course of the school year. Importantly, the teachers who took part in the network changed their own views of themselves—prior to taking the online course many teachers believed they could not be good at mathematics, and that mathematics was a set of procedures. As teachers changed these ideas about themselves, and about mathematics, they were able to teach differently. One of the teachers reflected on this personal change saying,

> "I thought it was going to be great for the kids, I never expected it to change me, that's been my greatest revelation in all of it."

By the end of the school year the students of the teachers in the network achieved at significantly higher levels on the mathematics portion of the CAASPP. The focus on mindset particularly raised the achievement of girls, language learners, and economically disadvantaged students (see Anderson et al., 2019). A survey taken by over 400 students showed that students significantly changed their beliefs, particularly changing their view that only fast thinkers could be successful, and their belief that only some people could be successful ($t = -8.69$, $p < 0.001$).

Teachers reflected that changed classroom environments—those that valued struggle and multi-dimensional mathematics—deeply and positively impacted their students:

> "The kids were thrilled, going 'Oh my gosh, he's doing it like that? It's OK that we struggle? It's OK we think differently?'"

> "I just want you to know this has meant a lot. Seeing how positive the kids are about their learning now has made a world of difference. The confidence they have is unlike anything I have ever seen."

Notably, the teachers also shared that the change in their teaching had started with a change in their own relationship with mathematics.

Analyses of the impact of the blended professional learning highlighted the importance of the combined attention to mindset (valuing brain growth and struggle), and to mathematics—working with teachers to open-up tasks and value multidimensional work (visual, numerical, verbal, modelling). The time that teachers were given to work together, access online and face-to-face professional development, and experience creative mathematics themselves, was critical to the success of the network. The blended approach and the details of teacher and student change is explained fully in Anderson et al., 2019.

Structured Coaching

The central goal of mathematics coaching is to support mathematics teacher learning and do so embedded in the contexts in which mathematics teachers do their work. Coaches can engage individual teachers and groups of teachers in a variety of potentially productive activities (Gibbons and Cobb, 2017), such as co-planning, examining student work, modeling instruction, and side-by-side coaching. In each, the teacher and coach co-participate in some way in the work of teaching—preparing for, enacting, or reflecting on it—and work together to make sense of mathematics content, student thinking, and pedagogy. For coaching to support teacher learning, teachers and coaches must make visible what they are noticing (Sherin et al., 2011), how they interpret what they see, and how and why they are making pedagogical decisions (Horn, 2005; Loughran, 2019).

Instructional coaching best contributes to school-wide mathematics instructional improvement when it is used as a tool to support the *collective* learning of teachers (Gibbons, 2017). In other words, the characteristic of effective professional learning that “provides coaching and expert support” does not stand alone; designating a “good mathematics teacher” as a coach has not proven to improve teaching practice by itself. Coaching is effective when it is structured to provide more than a model/co-teach/you teach feedback loop: “Coaches need to engage teachers in fundamental dialogue about mathematical content, mathematical learning, and student understanding” (Campbell and Griffin, 2017). Thus, coaching is effective when it is part of a broader professional learning plan that incorporates most or all of the other characteristics of effective professional learning, as in the following vignettes.

Coaching Vignettes: Making Sense of Content, Student Thinking, and Pedagogy

Grade Levels: Elementary Grades One, Two, and Four

Focus: Supporting the learning of practicing mathematics teachers within their teaching environments

Source: Jen Munson, Assistant Professor, Northwestern University

Each vignette below provides a brief example of three types of sensemaking—making sense of mathematics content, student thinking, and pedagogy—through and within mathematics coaching, drawn from data from a research study on effective mathematics coaching (Munson, 2018b). Each case involves a coach working one-on-one with a teacher, but sensemaking like that illustrated here can occur with a coach working with groups of teachers in much the same way.

Making Sense of Content: Co-planning for Joining and Separating Whole Numbers

Carmen, a 17-year veteran elementary teacher, had a goal of making mathematics more engaging for her second graders by incorporating rich tasks that required them to make sense of concepts. To choose or design such tasks in the unit she was teaching at the time, first Carmen needed to understand the mathematical concepts involved in joining and separating multi-digit numbers and strategies for doing so beyond the traditional algorithm she had been taught as a student. She began to read a text for elementary mathematics teachers about the ideas within joining and separating numbers (Van de Walle et al., 2012) with another second grade teacher in her school. She tried out various mathematical tasks in the text herself to understand how different strategies and representations worked. She then turned to her coach to discuss what she had been reading, the ideas that were exciting or confusing her, and how these might translate into what students might or could do.

In co-planning, Carmen met with her math coach, and they first focused on making sense of the joining and separating strategies describing in Carmen's professional reading. Carmen shared the strategies from the text she had tried to use herself and what she learned from those attempts. One thing Carmen found surprising was using addition to solve a problem that was written as subtraction. For instance, Carmen said that it had never occurred to her to solve problem like 34 – 27 by adding on to 27 to reach 34. As Carmen and her coach talked, they explored how closely coupled addition and subtraction are conceptually, so much that one never has to subtract, because every subtraction problem can be conceived of as a missing addend problem. Because Carmen's own schooling had rigidly separated addition and subtraction problems, she was surprised and delighted to see ways of breaking down this barrier.

Carmen then shared with the coach strategies that she found confusing or non-intuitive to use herself. In particular, Carmen was struggling to use the open number line as a tool for adding or subtracting. She had never thought visually or linearly about joining and separating numbers, and doing so without prerecorded markers made this strategy feel as open-ended as the number line itself. Carmen and the coach discussed how to think spatially about numbers so that joining and separating could decompose the number line into a series of hops from one point to another. The coach modeled her own thinking about how the number line represented a way of thinking about joining and separating as distances rather than digits. The coach gave some examples of how she thought through problems like 34 – 27 as hopping up from 27 to 30 and then from 30 to 34, using the decade number as a stopping point to decompose the distance between 27 and 34. Carmen and the coach tried this way of thinking together, and the coach pointed out that many children also conceive of numbers as distances and this model can be supportive of their reasoning about joining and separating, even if it was less intuitive to Carmen.

Near the end of their conversation, Carmen and her coach bridged from reasoning about the content to considering how her new thinking could look in her teaching. They discussed the kinds of tasks Carmen might try with her students to open up space for them to invent strategies for joining and separating numbers. One key idea that emerged was the use of context to support students' sensemaking; rather than giving students purely numerical tasks as she had done in the past, Carmen and her coach designed story problems that involved joining or separating so that students could—and needed to—interpret the situations and develop their own strategies.

In this example, co-planning was a key activity for the teacher and coach to have time to move between making sense of professional readings, mathematical concepts, strategies, and the pedagogical implications of each. In their conversation, the teacher and coach grounded their discussion in Carmen's goals, and the shared expertise of the text, the teacher, and the coach, each of whom brought important ideas and had a hand in making sense of content in a way that informed Carmen's teaching.

Making Sense of Student Thinking: Clinical Interviews about the Meaning of the Equal Sign

Quinn, an early career first grade teacher, was nearing the end of a unit on addition, subtraction, and the meaning of the equal sign with his students. In this unit, he challenged students to make sense of equations, finding missing values to make equations true, and determine whether an equation was true or false. Quinn's coach had been involved in co-planning some of this unit with Quinn and was present in the classroom during teaching some days to observe and talk with Quinn about what she noticed about student thinking.

Quinn launched each day's lesson with a number talk, and afterward, students typically worked in partners playing games that challenged them to make sense of equations. Some students had been very vocal throughout the unit, explaining their own reasoning and revoicing one another's thinking. But Quinn had come to feel that his sense of what the class was learning was driven by some – not all – students' participation. Some students had been absent, while others were simply more quiet. Quinn's assessment was that his students were learning and moving toward his goals for this unit, but she was uncertain if this was true for all students.

To get a more complete picture what students had learned, and what they still needed to learn in this unit, Quinn and his coach decided to conduct clinical interviews together with targeted students while the class played equation games. A clinical interview involves asking a student to do carefully chosen mathematical work and discuss their thinking along the way with an interviewer, with the goal of learning more precisely what the student understands. Quinn and his coach decided that interviewing Quinn's quiet first graders would give them better information than a written assessment, allowing them to ask follow-up questions and probe for reasoning. They selected four students from whom Quinn wanted to learn and designed two brief tasks to give them one-on-one: one involved finding the missing part (13 + __ = 18) and the other determining if an equation was true or false (15 – 5 = 13 + 2). From these two tasks they hoped to learn how students understood the meaning of the equal sign and how to use it to determine equality.

The coach and teacher sat together on the carpet with one student at a time, and Quinn led the interviews, presenting each task in turn to the child. As the student worked with manipulatives and a whiteboard, Quinn and the coach each asked probing questions to understand how the student was solving the problem, what reasoning the student used, and how they could articulate both their process and reasoning. During the interviews, when students became overwhelmed, the coach stepped in to modify the task so that the student could still show what they understood. For instance, when Amber froze upon seeing 13 + __ = 18 and said she couldn't do that because the numbers were too big, the coach changed the task to 3 + __ = 5 so that the numbers were not a barrier, and the teacher could still learn how the child made sense of a missing addend and the equal sign. At times during the interviews, Quinn expressed confusion about what a child was doing or thinking. At these moments, the coach either paused the interview to talk with the teacher about what they were noticing and how to interpret the student's thinking, or asked the child additional questions to try to elicit their thinking to make it clearer.

Between the individual interviews, Quinn and the coach discussed what they had learned about how the students were thinking, what they understood, what they were ready to learn, and what opportunities to learn they might need next. They found some trends. Some students needed more opportunities to count objects to build one-to-one correspondence above 20. They all could make sense of the equal sign as having the same value on both sides, but needed more experience with equations with expressions on both sides (such as the true or false task: 15 – 5 = 13 + 2). Some students could find a missing addend when the task was in context (e.g., Thirteen children were on the playground. Some more kids came. Now there are 18 kids on the playground. How many kids came?), but not when it was in an equation (13 + __ = 18). After the lesson, Quinn and his coach talked about an instructional plan to meet the needs of the students interviewed, along with the class as a whole, during the remainder of the unit.

In this example, the coach and teacher interacted with students about their thinking during mathematics, and in doing so they were able to gather, notice, and interpret student thinking in real time together. This allowed both the teacher and coach to make sense of student thinking grounded in the evidence they both generated in the interviews. So often, teachers are left explaining what students did, thought, or understood to a colleague after the fact, someone who did not co-witness the events and did not have the opportunity to notice student thinking themselves. The coach in this case positioned themself in the classroom with the teacher and his students to support both the gathering of formative assessment data and the interpreting of student thinking. As with the previous vignette, this collaborative work was a gateway to planning future instruction.

Making Sense of Pedagogy: Side-by-side Coaching during Conferring

Jane, a fourth-grade mathematics teacher leader, had built routines in her classroom around mathematical inquiry, in which each day students were given a task in context that they did not yet know how to solve. Students were asked to grapple with this task in small groups using strategies, models, and materials of their choice. During this collaborative work time, Jane circulated, conferring with the groups about their thinking and supporting their inquiry (Munson, 2018a). However, Jane felt she could learn more about how to use conferring to support students' mathematical thinking, and she accepted an invitation from her coach to work together on this pedagogy in the classroom.

For four weeks, two days each week, Jane and her coach engaged in side-by-side coaching to support Jane's goal of learning a pedagogical practice, conferring. Each day followed a similar pattern: Jane and her coach touched base briefly at the start of each lesson, Jane launched the lesson, they conferred with students together, Jane ended the lesson with a whole class discussion, and Jane and her coach debriefed what they had learn about pedagogy and from students that day.

During side-by-side coaching, Jane and her coach conferred with students together, moving throughout the classroom, side by side, to talk with students about their thinking. At times Jane led interactions with students while the coach observed, while at other times the coach modeled conferring or they co-led interactions. Throughout the four weeks, they focused on various parts of conferring and the thinking and decision-making that accompanied them. They worked together on (1) how to elicit student thinking and what features of student thinking to attend to, (2) how to interpret student thinking, particularly thinking-in-progress which can be challenging to understand, (3) how to decide what students need next to advance their thinking, and (4) what moves the teacher could use to help students move their thinking forward. They accomplished this by enacting the pedagogy together, talking through the myriad decisions that Jane needed to make in the moment to uncover, understand, and respond to her students' thinking.

By threading together teaching, professional development, and professional discourse, Jane's classroom became a rich site for teacher learning during teaching. Jane learned to slow down her interactions with students, give more time to eliciting student thinking, focus on ensuring students deeply understand the context of the tasks they were solving, and issue fewer directives to students, instead allowing them to make more mathematical decisions.

In this example, side-by-side coaching, in which teaching and professional learning happen together in the classroom, supported the teacher in making sense of a particular pedagogy. Instead of talking in the abstract, working on this pedagogy together in the classroom allowed the teacher to see and experiment with pedagogical moves with her own students within the lessons she had designed.

Closing Thoughts

It is worth noting that in each of these vignettes, the teachers' own goals for professional learning shaped both *what* the teacher and coach worked to make sense of—content, student thinking, or pedagogy—and *how* they worked together. Effective coaching aligns the teachers' goals with coaching activities that allow the teacher to actively make sense with a knowledgeable colleague.

Teacher Leadership

Ultimately, successful development and implementation of effective professional learning for teachers relies on expertise, which requires district capacity. However, the use of outside expertise can, over time, diminish the district's capacity to build internal leadership. Conversely, using in-house personnel that may lack the necessary expertise is not effective for creating lasting, meaningful changes that students are entitled to received. Districts must consider ways to build teacher, curricular, and administrative leadership, with the assistance of outside sources, to strengthen their long-term capacity to improve mathematics learning. Every district will have some teachers who show more interest in and more action around seeking opportunities to develop personal capacity to provide authentic mathematics learning opportunities. Identifying these "early adopters" and supporting their learning—as well as leadership roles in supporting other teachers—can be an effective way to strengthen a school or district's professional learning networks for mathematics.

This section begins with the development of teacher leadership as a core strategy for supporting improvement in teaching and learning, because research indicates that leadership and support are required in order for professional learning experiences to be turned into changes in teaching and learning practices (Lieberman and Miller, 2008; Weiss and Pasley, 2009). Teacher leadership is associated with increased teacher learning and creating collaborative professional cultures (York-Barr and Duke, 2004; Werner and Campbell, 2017), as well as being positively related to increased student achievement (Waters, Marzano, and McNulty, 2003).

Teacher leadership addressed in this section resonates with a definition of leadership from Julian Weissglass (1998), which states: “Teacher leadership is about taking responsibility for what matters to you.” In other words, teacher leaders include every teacher—those who are seeking or are designated teacher leaders, department chairs, teachers on special assignment, mentors and coaches, etc. Everyone has the capacity for leadership, and one goal of mathematics teacher leadership is to have many, rather than a few, people leading creatively every day and in all aspects of their lives (Kaser, Mundry, Stiles, and Loucks-Horsley, 2013). This view of teacher leadership differs from the traditional view in that leadership is not about power and authority. Instead, it embraces five practices of exemplary leaders (Kouzes and Posner, 2003), as listed in Figure 10.4.

Figure 10.4: Practices of Exemplary Leadership

PRACTICES OF EXEMPLARY LEADERS	DESCRIPTOR
Challenging the process	Searching for opportunities to change the status quo and innovative ways to improve
Inspiring a shared vision	Seeing the future and helping others create an ideal image of what the organization can become
Enabling others to act	Fostering collaboration and actively involving others
Modeling the way	Creating standards of excellence and leading by example
Encouraging the heart	Recognizing the many contributions that individuals make, sharing in the reward of their efforts, and celebrating accomplishments

Leadership development requires explicit attention, clear expectations, and resources, time, and expertise (Hopkins, Spillane, Jakopovic, and Heaton, 2013; Yow and Lotter, 2016). Mathematics teacher leaders need to continually build their: (1) in-depth understanding of the mathematics content and practices of the CA CCSSM; (2) thorough knowledge of the best practices in teaching and learning based in authentic contexts and problems; (3) understanding of school culture, organization, and politics; (4) understanding of change theory; (5) knowledge of how adults learn; and (6) practices that embrace continuous improvement. Additionally, leaders need skills in facilitation and communication, using data and decision making, and organization, to name a few.

Teacher leaders can take on a variety of roles to help colleagues and other educators, as well as parents, guardians, and community members become more aware of and aligned with improvements in mathematics teaching and learning.

These roles include leading in the areas of (1) instruction and assessment; (2) curriculum and instructional materials; (3) school culture that is supportive and proactive for the implementation of the CA CCSSM (4) community support and advocacy for active, authentic mathematics instruction; and (5) mathematics classroom implementation of the California ELA/ELD Standards. An explicit current in all of these roles must be access and equity for all students.

To develop these knowledge and skill sets, teacher leaders need professional learning targeted toward leadership. Learning experiences are most productive when they occur over time, provide feedback, are anchored in the practice of instructional leadership, and ground the leaders in mathematics practices and content (Fullan, 2015; Kaser et al., 2013; Darling-Hammond, Hyler, and Gardner, 2017). Districts need to develop leadership programs that embrace these attributes, and/or encourage their teacher leaders to participate in these types of leadership experiences through programs such as the CMP, the Silicon Valley Mathematics Initiative, Youcubed, and the California Mathematics Council.

Teacher leadership can be manifested in many forms, including presenting (at the school site, district, or professional organization level), consulting (as informal specialists for other mathematics teachers), facilitating (site-level department collaboration, Lesson study groups, and district-level efforts such as assessment and vertical alignment choices), and coaching.

The extensive literature on teacher leadership cited in this section provides additional sources for further learning by those seeking to empower and support teacher leaders.

Governance and Administrative Leadership for Professional Learning

School boards, working within their responsibilities, play an important role in supporting administrators and teachers to increase instructional knowledge and skills. When the board aligns its governance responsibilities and focuses on goals to increase students' mathematical understanding and success, district structures and resources strengthen administrative leadership.

Administrators play a key role in helping create and sustain a multi-layered system of support for teachers in their pedagogy and professional learning. There are several dimensions to the types of specific support administrators can provide, including having well-informed conversations about teaching and assessment, as well as feedback on instruction and critical conversations about instruction.

Together with their teaching staff and paraeducators, administrators may need to seek opportunities to understand more about the nature of mathematics learning and teaching presented in this framework. Leadership beliefs regarding mathematics instruction should be revisited, in consideration of the guidance presented in this framework, before becoming directives. For example, maintaining beliefs such as "fidelity to the curriculum" can undermine the focus and coherence called for in Chapter 1. It is critically important that clarity about focus, coherence, and rigor in mathematics be communicated at district, school and department levels. Addressing policies and practices around course offerings, placement, and de-tracking are essential conversations to be had at all levels. Unlike teachers, administrators are in the unique role to support and enact changes on a program level, rather than focus solely on a classroom level. Administrators should provide support for discussions on district and school-wide changes in practices and policies that can result in more equitable mathematics learning outcomes for all students. In establishing and maintaining regular communication with teachers about their teaching, their students, and the curriculum, administrators play a pivotal role in the confidence and vision necessary to help teachers explore new ways of ensuring all students can engage with mathematics. The guidance presented in this framework can serve as a starting point in helping to structure these conversations.

Administrators should be aware of this framework’s responses to the challenge posed by the principle of coherence. They are: **progressions** of learning across grades (thus, grade-band chapters rather than individual grade chapters), **big ideas**, and **relevance** to students’ lives. In particular, the learning progressions chapters (Chapter 3, 4, and 5) highlight the value in building powerful ideas about numbers and data that, over time, grow in meaning and resonate in subsequent grades’ topics; and on focusing learning upon productive habits of mind such as exploration, discovery and communication involving mathematics.

Administrators should be aware of the general principles guiding the development of the grade-band chapters (Chapters 6, 7, and 8). In general, these principles include: designing lessons from a small number of big ideas in each grade band; a preponderance of student time spent on authentic problems that engage multiple content and practice standards situated within one or more big ideas; a focus on connections, both between students’ lives and mathematical ideas; and strategies between different mathematical ideas of various topics across grade level.

Working with their teaching staff, administrators may need to identify opportunities to learn more about inclusive teaching strategies. Chapter 2 sets out the important qualities of mathematics classrooms that encourage student engagement and equitable outcomes. Through professional workshops, conferences, or other professional learning, administrators can support their teachers in this important learning. Partnerships with parents, families and caregivers can also provide valuable opportunities for administrators to rely upon as they work with teachers in addressing the totality of students' learning experiences. Family partnerships and experiences, especially those that are culturally and linguistically diverse, can create rich avenues of professional learning for teachers and teacher leaders. They should also draw upon teacher leaders at their school site or within their district who can provide support and knowledge of inclusive teaching approaches, especially those that focus on students who are culturally and linguistically diverse learners and students with learning differences. An important idea conveyed in this framework is that all students deserve access to high-level mathematics curriculum. Administrators are urged to read all of Chapter 9, especially the Introduction and the section Teaching Multidimensional Mathematics through Big Ideas and Connections, as they engage in conversations with teachers, school boards and parents on the ramifications of acceleration and tracking, and work with these same groups in careful consideration of the many alternatives which afford better access to higher level mathematics for all learners discussed in Chapter 9. In particular, Chapter 9 also elaborates on the Math Placement Act and provides a wealth of alternatives to tracking for administrators to consider.

This framework recommends that all students take the same, rich mathematics courses in kindergarten through grade eight. The chapters describing high school pathways and data science set out a structure for high school that will be new to many administrators, including the provision of a pathway in data science and statistics that can be taken as an alternative, or in addition, to calculus. This pathway should be open to all students, not only those who have been selected as mathematically oriented in younger grades. The provision of real data, and the encouragement of students to ask their own questions of the data, has the potential to broaden participation and make Science, Technology, Engineering, and Mathematics (STEM) pathways considerably more equitable. As new courses are developed and introduced into schools, it is important that administrators hold equity as a guiding principle and work to encourage equitable participation in the new courses.

The instructional vignettes in the framework can guide administrators to develop an awareness of the different teaching strategies and classroom conversations that provide opportunities to improve professional practice, and reflect upon the ways they can nurture these types of experiences for their mathematics teachers. The vignettes highlight the central role of classroom discourse and rich, open tasks in teaching and learning mathematics. One key perspective for administrators to recognize is that standards-driven instruction does not mean that each task results in learning of a single standard—in fact, multiple standards can often be learned through engagement with the rich tasks with multiple access points called for in Chapter 2; and mastery-based assessment at the “big idea” level (as described in Chapter 12) helps to reinforce the experience of mathematics as a sense-making, relevant activity. Administrators who understand that exploring a big idea through a single, rich task that provides opportunities for students to communicate their thinking with their peers and their teacher also understand that this often results in multiple standards learned, or reconnected with, in ways that foster both positive disposition toward mathematics and learning that lasts.

Additionally, administrators must acknowledge the inequities often perpetuated through traditional assessment strategies in the mathematics classroom, and how these assessment approaches can be re-envisioned (as described in Chapter 12) to provide a balanced approach in assessing the effectiveness of mathematics instruction. Administrators should look critically at program data to determine how systems are supporting or inhibiting access to equitable mathematics. Transcript analysis and course-taking patterns, correlated with metrics of achievement provide a broader view of student success than solely focusing on exam achievement. The results of multiple assessment strategies—rather than a single score on a test—reflect a more complete understanding of student learning. Standards-based assessment provides an approach to grading that focuses learning on standards and mastery rather than emphasizing grade ranges or percentages. Broadened approaches to assessment in a district/school often mean that administrators prioritize participation in ongoing professional learning on the topic of mathematics education and assessment of learning. Administrators leverage their understanding and use of the Multi-Tiered System of Support (MTSS, CDE, n.d.) by supporting teachers in aspects of MTSS implementation such as integration of instruction with intervention and a focus on continuous improvement.

Several ways that administrators can help support and incentivize effective professional learning are outlined in "Effective Teacher Professional Development" (Darling-Hammond, Hyler, and Gardner, 2017):

1. Since a critical component of rich learning is the planning time and pedagogical knowledge necessary to facilitate an active mathematics learning environment, administrators should prioritize time for professional learning and collaboration when designing schedules. Professional learning communities, peer coaching and observations across classrooms, and collaborative planning all provide important opportunities for educator learning.
2. Periodic needs assessments (at school or district level) use staff surveys to identify areas of professional learning most needed and desired by educators. This helps ensure that professional learning is connected to practice and makes impact on practice much more likely.
3. District and school administrators should identify and develop expert teachers as mentors and coaches to support the professional learning of other educators. These "expert teachers" need their own support, structure, and professional learning in order to be effective.
4. Districts and schools should ensure that professional learning opportunities are integrated with efforts to implement legal requirements, such as the Every Student Succeeds Act (ESSA) school improvement initiatives. Mandates, such as the use of data to inform instruction and the creation of positive and inclusive learning environments, are primarily effective only when educators experience them as supportive of their improving classroom practice, as opposed to compliance exercises that add more paperwork to busy days.
5. In order to address professional learning needs of rural communities and to develop intra-district and intra-school collaboration, Titles II and IV of ESSA should be used to support technology-facilitated opportunities for professional learning and coaching.

6. District and school administrators can seek out funding which supports professional learning opportunities and connect this to continuing education units. These opportunities can include many of the types listed below, such as institutes, workshops, mathematics-specific conferences, and seminars, and also sustained engagement in collaboration, mentoring, and coaching. Possible funding sources include Local Control Accountability Plans, state and federal grant programs, community/business partnerships, and foundations.

Some specific tools to aid instructional leaders in supporting quality mathematics instruction include organizations that are available to partner with schools, as well as observation and planning guides.

These organizations and tools enable administrators’ critical role in conveying high expectations for mathematics instruction—expectations made attainable by providing teachers with resources, including time for planning lessons, professional learning, and collaboration—with a focus on and aligned to agreed-upon school-wide priorities and strategies. Administrators can provide constructive, informative feedback that builds on teachers’ strengths, while the teachers implement their plans. Frequent discussions about mathematics teaching and collaborations around mathematics lessons can allow the school administrator to engage teachers in productive conversations and provide relevant feedback on instructional practices. The general observation pattern in many California schools—where a classroom teacher is observed formally once a year—is insufficient for educators to gain an understanding of, and support, teachers’ instruction. Scheduling frequent and sustained interaction with teachers improves an administrator’s engagement with students and teachers, and allows them to glean a more complete picture of the instructional practices used by their teachers and which supports are needed to bring about positive growth.

Role of Parents, Guardians, and Families

While the school classroom is a primary learning environment for mathematics education, home and community also play significant roles. Through involvement at every level, parents, guardians, and families can motivate students to develop a lifelong appreciation of mathematics learning. Families can also provide a supportive home setting for students to learn and prepare for school. Partnering with parents, guardians, and families in understanding and supporting authentic mathematics education and active learning pedagogy is key.

A substantial body of research asserts that “effective family engagement depends on the close working relationships between teachers and each child’s family (Niebuhr, Arseo, and Simeon, 2021) and that these relationships require building of capacity for families and educators. As they have during the global pandemic of 2020-21, families can support learning as “co-creators, supporters, encouragers, monitors, advocates, and models” (Mapp and Bergman, 2019). Families are key in supporting the development of future mathematicians by increasing students’ confidence, developing a growth mindset, and providing examples of math applied to real-life situations. Creating a bridge between children and their families helps children to deepen their connection to their learning and to be more successful academically.

The passage below from *Black, Indigenous, and Latinx Parents as Partners in Mathematics Education* by TODOS: Mathematics for ALL (2020) provides insights about the assets parents bring when invited into the teaching and learning process:

> Black, Indigenous, and Latinx parents have a lot to offer classrooms; however, they are not always asked to join and be a part of the instruction. Ishimaru, Barajas-López, and Bang (2105) has argued for the involvement of parents from nondominant groups in schooling not as passive recipients of knowledge, but as “expert collaborators and fellow leaders.” (p. 14). Given our current expectation of online and hybrid classes, schools can develop an online learning culture leveraging school/home connections that support mathematics identity and agency for students and parents. Research on Latinx parents visiting classrooms suggests that observations and debriefs of classroom visits were one way that parents were able to both reflect on ways to support their students and develop leadership in mathematics education (Civil and Menéndez, 2012).

Because the CA CCSSM and this framework present mathematics instruction that is significantly different than what many parents experienced as students, it is critical to educate parents and guardians about what to expect and about the reasons and research behind the changes. Educating and engaging parents and guardians should include opportunities for them to experience rich, authentic, culturally-sustaining mathematical tasks in active-learning ways (including support for parents who speak languages other than English), not simply written descriptions of it. Validating and valuing parents', guardians', and families' central contributions to education is enhanced when they have opportunities to use their own language, culture, and knowledge through relevant experiences rooted in the school context.

Furthermore, parents and guardians who become more knowledgeable through such an experience can more effectively support students' learning beyond the classroom. Parents and guardians can monitor their student's progress not just for content knowledge, but for understanding of and engagement in mathematical practices or a developing inclination to use mathematics to make sense of their world. Parents and guardians can also foster social interactions (e.g., by providing support for collaborative classroom or out-of-classroom projects) and become involved in educational activities promoted at the school site (e.g., math fairs and math clubs).

A model to support the development of family and school partnerships is the National Parent Teacher Association (PTA), which has developed standards for Family-School Partnerships. These standards focus on several aspects of the partnership, providing recommendations on how to foster effective communication and trust to support students' success. In addition to the standards, the National PTA has developed a guide that provides a rubric with examples for what family-school partnerships look like at the emerging, progressing, and excelling levels. Parents, guardians, families, and school leaders may want to use these examples to evaluate and enhance the family-school collaboration at their school site. Specifically, involving parents who have a background in mathematics (including in such areas as the building trades and cooking, as well as more traditional STEM areas) will help develop partnerships with the community that can provide much-needed support for classroom instruction.

The California *ELA/ELD Framework* provides specific suggestions for parent, guardian, and family involvement when those families speak a language other than English or are new to the United States. When possible, having parents who have experience with mathematics and speak a home language that students also speak would be a great support for the parents of those students who are not as experienced with mathematics (CDE, 2014, Chapter 11).

Conclusion

A broad system of support to enable all students to succeed in their mathematics learning consists of many interconnected parts. Teachers, as the drivers of learning, continually refine and adapt their practice to address the many dimensions in creating a rich mathematical learning environment focused on active learning for all students in their classrooms. By supporting teachers with the resources, time, insight, and encouragement to become ever-more effective practitioners of their craft, administrators serve a critical role in the system. The elements for effective professional development described in this chapter provide administrators and other stakeholders with guidance on creating high-quality learning experiences for teachers, and the examples listed are a small sampling of the variety of professional development experiences available. Supporting teachers, both in their own learning and in their teaching, ultimately supports the students who rely upon these teachers.

Long Description for Chapter 10

Figure 10.3: Bar chart showing an improvement among teachers in knowledge of each of the Five Mathematical Mindset Practices by practice and number of teachers.

Practice 1

Level	Beginning of the Year	Middle of the Year
Beginning	10	1
Developing	12	9
Expanding	5	17

Practice 2

Level	Beginning of the Year	Middle of the Year
Beginning	14	1
Developing	7	11

Level	Beginning of the Year	Middle of the Year
Expanding	6	15

Practice 3

Level	Beginning of the Year	Middle of the Year
Beginning	10	0
Developing	14	19
Expanding	3	8

Practice 4

Level	Beginning of the Year	Middle of the Year
Beginning	11	1
Developing	12	16
Expanding	4	10

Practice 5

Level	Beginning of the Year	Middle of the Year
Beginning	4	0
Developing	9	12
Expanding	14	15

Return to graphic.