Mathematics Framework
Chapter 9: Structuring School Experiences for Equity and Engagement

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Introduction

The previous chapter described ways in which courses can be sequenced to offer all students access to high level mathematics content. This chapter describes methods of teaching that enable all students to be appropriately challenged, without requiring that all students work on the same mathematics or be placed in inflexible course sequences that make it difficult for them to move into or between STEM or non-STEM pathways if they so choose. The goal is to expand access to rigorous mathematics for all students, allowing each to experience the joy and excitement of well-taught mathematics in ways that stimulate their learning and engagement.

Expanding Access to Rigorous Mathematics for All

As schools become increasingly diverse in terms of language, culture, socio-economic status, past experience, interests, and learning needs, it is important for California educators to carefully consider the best ways to enable all students to excel in mathematics. All students are different, a fact to be celebrated. The differing ways students think and work make teaching rewarding and interesting. Teachers of mathematics are accustomed to classrooms where students with a range of prior mathematics exposure offer different ideas and strategies for solving problems.

Some students grasp certain ideas more quickly, while others appreciate more time to think about and engage more fully with those ideas. These differences do not indicate students' degree of mathematics potential. Among mathematicians, some of the highest-level achievers report that they think slowly and deeply. Laurent Schwartz, who won the Fields Medal in mathematics, reflected on his school days with these words:

> I was always deeply uncertain about my own intellectual capacity; I thought I was unintelligent. And it is true that I was, and still am, rather slow. I need time to seize things because I always need to understand them fully. Towards the end of the eleventh grade, I secretly thought of myself as stupid. I worried about this for a long time.
>
> I'm still just as slow...At the end of the eleventh grade, I took the measure of the situation, and came to the conclusion that rapidity doesn't have a precise relation to intelligence. What is important is to deeply understand things and their relations to each other. This is where intelligence lies. The fact of being quick or slow isn't really relevant. (Schwartz, 2001).

Despite such high-profile examples of deliberative mathematical thinkers, it has long been a practice in mathematics education to value speedy thinking and fast memorization of facts. Yet deep understanding should be the primary goal of classrooms. It is deep understanding that allows people to apply mathematics, make discoveries, and expand mathematical learning. As explained in the previous chapter, mathematics experts and leading institutions of higher education have concluded that racing through mathematics without deep understanding is misguided, as it does not develop the mathematical foundation that is required for ongoing progress in quantitative fields. Moreover, students' opportunities for learning should never be limited by perceptions of their ability based on factors such as their gender, race, or language background (Chestnut et al., 2018; Fennema et al., 1990; Del Pinal, Madva, and Reuter, 2017; Elmore and Luna-Lucero, 2017; Tiedemann, 2000).

For many years there has been an assumption that people either are or are not born with a "math brain" (Doidge, 2007; Maguire et al., 2006). This does not mean that all people are born with the same brain; it does mean that abilities grow through the many opportunities students receive for brain development. The belief nonetheless persists that some people are innately “good” or “bad” in math or, for other reasons, do not belong in higher level math classes. Attaching labels to students that suggest fixed ideas about ability is unproductive, since such labels often lead to differential opportunities to learn that underestimate the possibilities for growth.

While this framework recognizes that some students are born with learning challenges and others with learning advantages, and that students have differing experiences and opportunities before they arrive at school, it also recognizes that no student’s mathematical ability is fixed. All students are capable of strong learning gains, given effective teaching and support that fosters a growth mindset. Many studies show that student lags in math performance, which may seem to signify difference in ability, can be changed through interventions (Kwon et al., 2021; Frontiers et al., 2007; Moses and Cobb, 2002).

Students should have early, ongoing, and equitable opportunities to develop their abilities. Mathematical excellence can develop or reveal itself at any life stage. Consider, for example, Nicholas Letchford, who started school labeled as having a low IQ and significant special educational needs. He went on to graduate from Oxford University with a doctorate in applied mathematics (Letchford, 2018).

This framework proposes grouping systems and other supports that keep higher level pathways open to more students for a longer time, while enabling high-achieving students to move more rapidly and deeply through content, as appropriate. The framework recognizes the diversity of student achievement and sets out ways to teach that ensure that all students receive appropriate support and challenge—including providing all students with challenging work rather than leaving some students bored or working at levels lower than what they may be capable of, which can happen if teachers require the entire class to stay together or learn the same content in the same way or at the same pace.

High-achieving students may be challenged in a variety of ways, including by engagement in ambitious inquiries in any given course, by engaging in additional mathematics learning opportunities in supplementary courses or extracurricular challenges, and/or by acceleration through a course pathway. When acceleration occurs, it should be in the context of enabling access for students who are clearly ready for more challenging content, rather than in the context of reducing the opportunities for other young people to access challenging content from which they could benefit—as can happen with such practices as tracking.

The remainder of this chapter sets out the different ways students may be challenged and supported in mathematics classes with examples of how districts and schools have enacted systems of grouping that support a wider range of students in accessing higher level content. If the goal is to open mathematics pathways to more students and give greater challenge to high achieving students to develop broader proficiency and long-term interest in quantitative fields, then this framework recommends reshaping the content that is offered to students—the way it is taught, and the organization of students learning the content—in the following ways.

Replacing Early Tracking with Adaptive Teaching and Flexible Student Grouping

Grouping strategies can benefit students when they are a means of providing high-quality instruction that meets student needs and broadens opportunities for future learning. Such strategies sharply contrast with traditional early tracking, which prescribes the future and closes down subsequent opportunities.

US schools have a long history of placing students in “tracks” for math instruction. Tracking systems were designed in the early twentieth century to place students on pathways through school. As with trains on tracks, student pathways led to different, predetermined journeys and destinations, in this case through subsequent years of education.

Approximately three quarters of US grade eight students are tracked in mathematics, a proportion that has not changed in many years (Antonovics et al., 2022; Loveless, 2021). For many, this tracking begins in the early years of elementary school—often around third grade—or at the beginning of middle school in fifth or sixth grade. Schools sometimes use elementary school test data to determine students' placement, which typically also determines their ultimate destination. Because students are then taught different content, they often cannot easily change pathways. This practice is unjustifiable. Educators cannot predict what a student can do in their later school years based on their proficiencies at the elementary level in mathematics or their English language facility at that time. Yet tracking is pervasive, unnecessarily limiting many students' future options.

Various definitions of tracking exist in practical usage, and the term "tracking" is sometimes confused with grouping, which allows students to receive focused instruction that meets their immediate needs at a moment in time, rather than setting them on a fixed long-term course (Antonovics et al., 2022; Betts, Zau, and Rice, 2003; Collins and Gan, 2013), However, students of color, recent immigrants, and those from low-income families have often been "tracked down" into less challenging, rote-oriented coursework. Such coursework is also generally less well-taught, in large part because these classes are often assigned to the least experienced and least expert teachers, which further restricts later opportunities (Bacher-Hicks and Avery, 2018; Reardon, 2019; Oakes, 2005).

Tracking of this sort has been frequently critiqued not only because it depresses the achievement of students in the lower track. It also often rations access to higher tracks for a set number of students on the basis of criteria that do not predict success in the more ambitious curriculum (e.g., Callahan, Humphries, and Buontempo, 2020; Grissom and Redding, 2016); Guyon, Maurin, and McNally, 2011; Kalogrides and Loeb, 2013; Oakes, 2005).

A meta-analysis of 15 studies on tracking, conducted in and outside the US, found that classes that offer a more ambitious curriculum to all students have tended to support improved outcomes for initially lower-achieving students, without negative effects for higher-achieving students (Rui, 2009). Another review of international evidence about tracking found that, while most Organisation for Economic Co-operation and Development (OECD) countries do not differentiate curriculum options for students in the early grades, those that track students into different schools or curriculum pathways in elementary school increase inequality in learning significantly (Hanushek and Woessmann, 2006; Woessmann, 2009). Woessmann (2009) concludes that “Early tracking leads to a systematic increase in inequality of student performance between the end of the primary and the end of lower-secondary school;” furthermore, while “later school tracking increases equality of opportunity, [it] is not associated with a lower performance level.”

Although no country’s approach directly translates to the context of another, there are common curricular approaches resembling those suggested in this framework that are taught in non-tracked classes across many of the highest-achieving nations in mathematics, including Japan, Korea, Estonia, and Finland (see Hemmi, Brating, and Lepik, 2020; National Center on Education and the Economy, n.d.; Okano and Tsuchiya, 1999; Stigler and Hiebert, 1997. See also chapter 8.). In keeping with approaches used in these and many other countries, the National Council of Teachers of Mathematics (NCTM) strongly advocates for creating a system of middle school mathematics courses that will “dismantle inequitable structures,” including “the practice of ability grouping and tracking students into qualitatively different courses” (NCTM, 2020). NCTM has argued that if the US is to regain its lost ground in mathematics, districts and schools must confront the structural inequities of tracking and ability grouping that restrict most students from accessing higher level mathematics and strengthen efforts to support all students in learning a common, rigorous curriculum.

While early tracking of students into low-level courses has been problematic, there is evidence that thoughtful grouping of students to ensure they receive high-quality instruction geared to their needs at a moment in time can be helpful. Such an approach can help students who need to fill gaps in their prior learning as well as high-achieving students who are ready for greater challenges. As noted in the earlier discussion of the New York de-tracking study (see chapter 8), additional math labs attached to more rigorous courses can also be a useful strategy for supporting mathematics learning.

In addition, successful strategies for teaching broader and deeper mathematics to heterogenous groups of students require attention to teacher learning. Teachers need support to rethink math teaching and acquire skills and strategies that result in the changes in practice required for teaching mathematics with multidimensional tasks to a wide range of learners. Districts and schools need to accompany these structural, curricular, and pedagogical changes with professional development and time for collaborative learning and planning. (See chapter 10 for more on teacher support.)

Productive Strategies for Teaching Diverse Students

A number of California districts are attempting to improve opportunities for all students to excel in mathematics through innovative approaches to grouping students that seek to ameliorate the negative impacts of long-term tracking. These approaches include:

Grouping students later and offering multiple junctures for acceleration. One form of flexible grouping involves moving the beginning of separate course pathways to later grades—e.g., from fourth, fifth, or sixth grade to at least eighth grade—and supporting extra course-taking options during the school year or during summer school so that students may accelerate at any time during middle or high school. Since student interest and engagement can fluctuate significantly during adolescent years, this approach enables more students to build a strong mathematics identity and skill set both before and after they are assigned to (or choose) a placement. Importantly, within the course pathways offered by a school or district, there should also be the opportunity for high-achieving students to accelerate at any time, when they are ready to do so.

Districts and schools should also factor student interest and desire into student placement. Many students blossom when they are offered higher level content, and they frequently choose to step up to challenges, especially when they have the support they need to succeed. Studies verify that such “tracking up” into more challenging classes can have benefits for students, and those benefits are particularly strong for students of color (see, for example, Card and Giuliano, 2016, who also found that high-achieving students of color are typically overlooked for these opportunities). Where separate pathways are used, students should be enabled to pursue additional study options at multiple junctures if they wish to shift the rate at which they progress.

Rethinking course pathways. It would be helpful for the state to convene a working group of mathematics experts to discuss and clarify possible high school pathways. Such guidance could help districts and schools reverse-engineer high school pathways so that advanced courses are attainable by students who begin with the default Algebra I or Mathematics I course in ninth grade, rather than eighth grade. As discussed in chapter eight, this is possible both because there is repetitive content in the current traditional pathway to calculus and because the California Common Core State Standards for Mathematics (CA CCSSM) middle grades expectations, which include a strong start on algebra, are sufficiently rigorous preparation for a four-year high school pathway that includes advanced classes such as statistics or calculus.

Providing additional support and expanded learning time. Students may benefit from support or co-requisite courses taken alongside their primary math class to help them gain deep understanding and mastery of important math ideas and to revisit content that may have been missed or poorly understood in previous years. They may also take more than one mathematics course a year in order to reach more advanced courses during their high school years. One approach is to offer summer classes—before high school and during high school summers—where students can take a course or strengthen their readiness for the next sequence of courses. The Algebra Project, created by Bob Moses, has designed curricula used in summer- and after-school programs, as well as during school-year courses, that enable students both to strengthen their skills and develop a strong mathematics identity. Similarly, The Calculus Project enables higher achievement for traditionally underrepresented students by working with schools to offer preparatory courses in the summer, as well as after-school study groups and tutoring during the school year to support mathematics instruction from grades eight through twelve.

In another example, Louisiana implemented a pilot program where high school students enrolled in two periods of Algebra I with the same teacher for both periods, using a curriculum that interwove foundational mathematics and algebra content (NCTM, 2018). The extended time—as well as additional supports for teachers—were critical in helping ninth graders successfully complete Algebra I. A number of studies have shown that academic support courses for high school mathematics can be effective in supporting more students to succeed in mathematics learning (see US Department of Education, 2018). Such courses can provide additional time for classroom instruction (as in the Louisiana pilot), homework support, and supplemental assignments that emphasize study skills and preparation in the core companion courses.

Some districts offer support classes that are open to everyone. In one California district, middle schools offered a class, open to all students, that followed the regular mathematics class (see Boaler, 2016). Though the content was the same as the previous class, the extra time allowed students to discuss ideas further and ask more questions. Many students—both higher and lower achieving—chose to enroll in the class. It is important that such classes be given positive names that characterize them as providing additional depth, not remediation.

Additional opportunities may also be provided outside of the regular school day. Such opportunities can provide experiences with mathematics that differ from those that students typically encounter in school—in particular, experiences that lend themselves to a more investigative approach. Two highly regarded examples include math circles and Math Olympiad classes. Both programs offer opportunities for mathematical problem solving for students at different grade levels as well as professional development for teachers (Math Circle Network, n.d.; Math Olympiads for Elementary and Middle Schools, n.d.). These opportunities for students to engage in mathematical problem solving outside of regular school hours are often highly successful, since they can help students develop a positive mathematics identity (Langer-Osuna, 2007, 2017) and broaden their view of what it means to do mathematics.

Structures that diverge from traditional course scheduling, such as double periods or block scheduling, can expand learning and instructional time, thereby allowing for the support students may need to master foundational skills and accelerate their learning.

Other time-expanding options include mathematics labs appended to courses that allow for more individualized, diagnostic instruction, tutoring, or small group instruction after school or in the summer (discussed below under Personalized Learning).

Providing personalized learning. Another strategy for attending to different achievement levels of students is to provide personalized learning. In this framework, personalized learning means learning experiences that are customized “for each student according to his or her unique skills, abilities, preferences, background, and experiences” (Herold, 2019). It can be provided both within individual courses and across course pathways. For example, teachers can allow students to work through courses at different paces, with decisions about advancement made on the basis of student work that demonstrates their readiness. (See the snapshot *Personalized by Teachers*, below.) Such personalized decisions about advancement are very different from group-based, long-term tracking decisions that predetermine how students will be processed through standardized coursework at a standardized pace. (See also personalized learning through one-on-one and small group tutoring, below.)

Personalized learning can be supported by emerging technology-based systems (see, for example, Murphy et al., 2014), which offer promise for helping educators meet the individual needs of learners across the achievement spectrum in heterogeneous classrooms (Deunk et al., 2018). When well-designed technology tools are used appropriately, they can allow students to work at their own pace on material they are ready to learn, with teacher and peer support (Phillips et al., 2000; Beal et al., 2007; Darling-Hammond, Zielezinski, and Goldman, 2014; J-PAL Evidence Review, 2019). Chapter 11 of this framework provides more information on integrating technology, and California’s Digital Learning Integration and Standards Guidance (CDE, 2021) may be a particularly helpful resource. Additionally, several organizations now offer online opportunities for targeted practice in particular mathematical topics.

Snapshot: Personalized by Teachers

A high school mathematics department wanted to tackle the problems of fixed tracking systems by giving students choice and allowing them to use their existing work in different courses as the basis for decisions about which courses they might advance to. The teachers arranged for students to take assessments at the end of each course unit, allowing them to move at a pace appropriate for them.

In this team-taught program, each student is assigned to a lead teacher who sets goals for the student and tracks progress. Students meet as a class with their lead teacher each day at the beginning of the period to work on open problems or participate in number or data talks. Students then transition to different rooms for each course (e.g., Algebra, Geometry, Algebra II, or Trigonometry), where they sit in groups and work on the course materials while the teacher circulates around the room, providing small group instruction, asking guiding questions, and keeping students on task. When students finish a topic, they submit a request to be assessed, which their lead teacher approves after checking that they have completed all of the materials for that topic. Students then take an assessment and, if they achieve a score of at least 70 percent, they are free to move on to the next topic. If they score below 70 percent, they work with their lead teacher to learn, understand, and be able to apply the material. Students also have the option to retake any assessment, regardless of their score, and teachers always accept the higher grade. Once students have completed all work from their current course, they transition directly into the next course. Multiple team-taught courses (such as Algebra, Geometry, Algebra II, and Trigonometry) run in the same period.

This approach has allowed students to exercise agency and to move ahead whenever they have learned the material for a course. Said one teacher:

> Some students who have always hated math have grown to love it because they are able to take control of their learning. They move at the pace that is right for them, and while it may be slower than in a traditional year-long class, they feel like they are finally learning the material, and their assessment scores show that. Other students have embraced the idea that sometimes you need to slow down, to build that strong foundation, in order to pick up speed later. Other students have set lofty goals for themselves and have a strong desire to complete multiple

> courses in one year. Given that they are demonstrating mastery on their assessments, we don't believe in holding them back. This is allowing students to have multiple pathways to higher level math courses. They are no longer limited by a placement decision most likely made in sixth grade. Students can still start high school in algebra and get to calculus or beyond if that is their goal.

(*end snapshot)*

The great benefit of personalized systems is that they allow students to work at their own pace on content that is appropriate for their understanding. To ensure that students experience the insights of others and engage in joint problem solving, individualized experiences can also be combined with opportunities for mathematical collaboration.

Among the most effective systems are those that combine different experiences—that is, where students divide their time between working on a computer and working with other students and the teacher on rich mathematics developing conceptual understanding. In one successful teacher-developed approach, students engage in blended, self-paced, mastery-based learning with teacher-made videos supplementing in-class problem-solving individually and in collaborative groups, with continual assessment and revision of work moving students toward confidence and competence (Modern Classroom, 2021). A similar model developed by a middle school teacher and now taught in many schools, uses diagnostic assessments to create a tailored set of assignments for each student that the teacher can use in technology-infused mix of direct instruction, collaborative work with peers, and individualized learning. A study of this model found that participating students improved at a faster rate, on average, on mathematics assessments than did a nationally representative comparison group (Margolis, 2019).

Yet another approach, which offers multiple strategies that can be used in individual or collaborative study to learn and practice content mapped to standards at each student's level of mastery, has been found to reduce math anxiety and to support greater achievement for students at different initial levels of achievement when used to complement classroom instruction (Murphy et al., 2014). Whichever approach is used, the goal should be to create a personalized learning environment that is focused on rich mathematics and through which students can conduct mathematical investigations and work on big ideas and mathematical connections.

For now, it is crucial that schools and districts considering personalized learning products or programs review them carefully to ensure that they:

- Develop mathematical concepts, problem-solving strategies (including computation), and applications in ways wherein each supports the other.
- Design student activities around big ideas that connect multiple content standards through engagement in the Standards for Mathematical Practice (SMPs) in the context of authentic investigation.
- Emphasize connections between mathematical ideas, strategies, and representations, rather than isolated skills.
- Include collaborative components in student investigations, to build mathematical content and practices that emphasize mathematical communication and discourse.

Including one-on-one or small group tutoring. Districts are increasingly deploying one-on-one or small group tutoring to help students secure skills that they may have missed or not fully mastered. A growing research base shows that specific programs offered by trained tutors with frequent, regularly scheduled sessions can result in substantial gains in mathematics achievement, allowing students to accelerate their learning and sustain a path to higher level courses (US Department of Education, 2017; Nickow, Oreopoulos, and Quan, 2020). Systematic use of tutoring could reduce the felt need for lower-track classes that derail students at an early age from paths leading to potential STEM careers.

The COVID-19 pandemic increased the use of both online and in-person tutoring options nationwide. In California, the state responded to the pandemic by providing resources to local school districts to promote learning acceleration and recovery, including for such acceleration strategies as summer school, expanded learning time, and the use of high-dose tutoring—i.e., tutoring that is delivered more than three days per week or at a rate of at least 50 hours over 36 weeks. A research overview on high-dosage tutoring provided by the National Student Support Accelerator reports that a meta-analysis of almost 200 studies found tutoring to have large, positive impacts on student achievement in both math and reading" (White et al., n.d.). Such tutoring may be particularly impactful for students from lower income families (Dietrichson et al., 2017).

High-impact tutoring programs tend to include the following characteristics (National Student Support Accelerator, n.d.):

- High-dosage delivery (at least 30 minutes at least 3 times/week)
- A stated focus on cultivating tutor-student relationships
- Use of formative assessments to monitor student learning
- Alignment with the school curriculum
- Formalized tutor training and support

Increasing Student Success with Multidimensional Teaching

As highlighted above and in earlier chapters, a number of schools, districts, and educational systems have worked to open pathways to high achievement to significantly more students by eliminating low level math classes and providing all students with deeper and broader math through multidimensional math teaching. Instead of teaching through narrow questions that engage some students but are inaccessible to others and leave still others bored and unchallenged, teachers focus on big ideas and connections. They teach through more open tasks that students can approach in different ways. Such an approach allows students to explore questions of interest and work on mathematics at different levels.

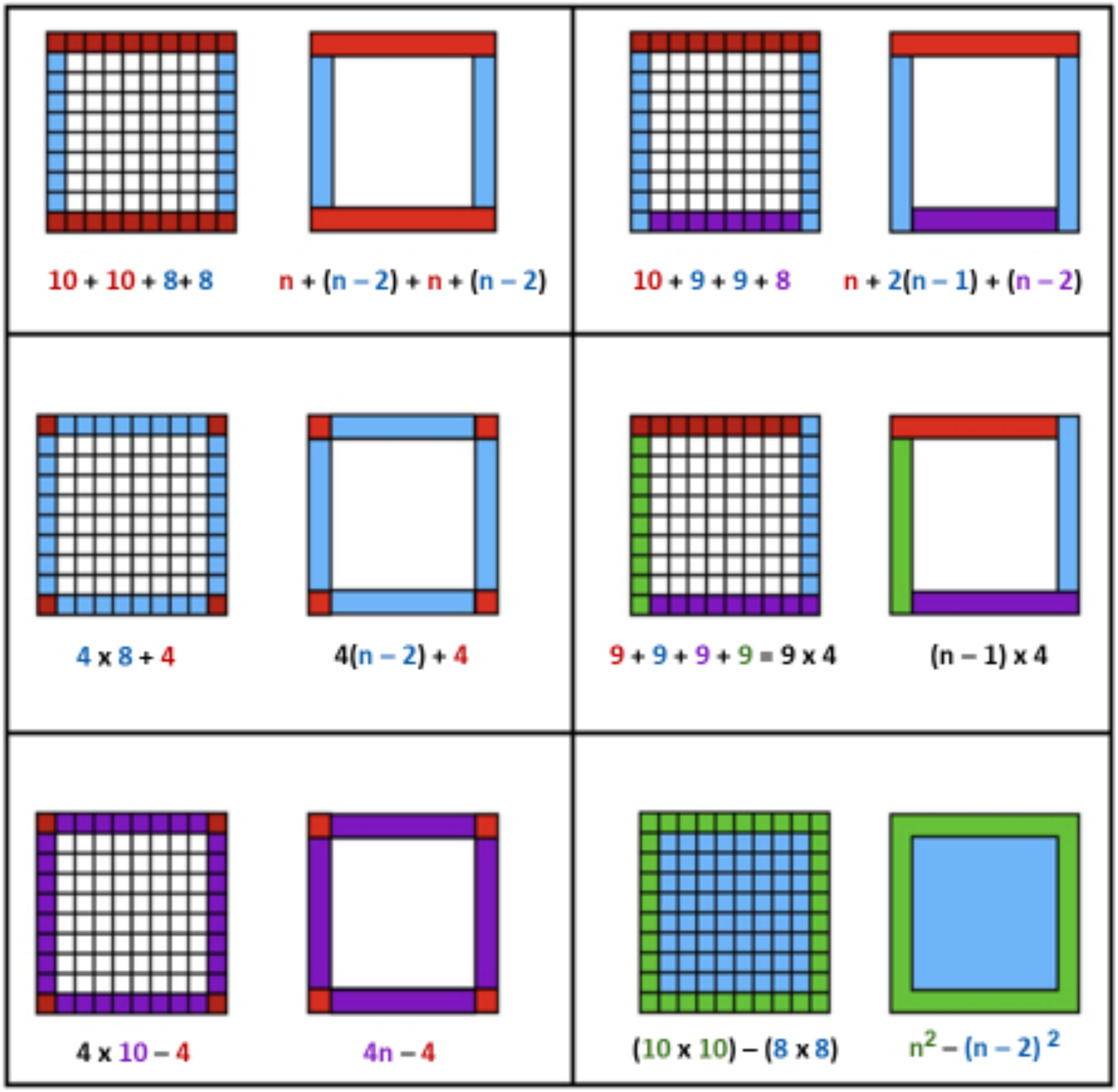
For example, in a typical algebra classroom students might be asked to simplify these expressions:

1. n + (n + 2) + n + (n – 2)
2. 4 (n – 2) + 4
3. n + 2(n – 1) + (n – 2)
4. 4(n – 1)
5. $n^2 - (n-2)^2$

In a classroom focused on big ideas and connections, the teacher may choose generalizing as a big idea and introduce the idea through the “border problem,” as explained in chapter seven. In this approach, students consider the tiles on a border of different sized squares, eventually describing the border size with words, and then algebraically, and then forming equivalent algebraic expressions. This is a more open task than the initial one, as it allows students to explore and make connections in multiple ways. It is also a task with a low floor and high ceiling—all students can visualize borders of squares, and higher achieving students can extend the problem to borders of different shapes. This task exemplifies multidimensional math teaching and also encourages the principles of UDL—students can engage with it in different ways, with visuals, words, numbers, and discussion, which leads to a deep understanding of generalization and equivalent expressions.

The initial task is one dimensional—students simplify expressions. The border task is multidimensional as students engage in many dimensions of mathematics—generalizing, visualizing and drawing, communicating, connecting words, expressions and visuals. Such tasks take longer than narrow questions involving equivalent expressions. But research has shown that a teaching approach geared to big ideas, with fewer but deeper and longer tasks, not only engages all students—whatever their prior achievement—but also increases understanding for all students, including the highest achievers (see also Nasir et al., 2014; Boaler and Staples, 2008).

Figure 9.1 The Border Problem



Source: YouCubed, 2018.

Long description of figure 9.1

An important resource for districts and schools that choose to offer higher level mathematics to all students are the textbooks and instructional resources designed to support teaching big ideas and connections. Textbooks that share deep mathematics tasks that can be worked on across a sequence of multi-day investigations are appropriate, as opposed to textbooks that offer short, closed questions, with limited interest or appeal to many students. In high school, truly integrated content provides greater opportunities for broad and deep tasks that provide appropriate challenge for all students. Studies of such curricula being used in urban, suburban, and rural districts, including in California, show that students have achieved at significantly higher levels on tests of problem solving, conceptual understanding, and applied mathematics and have enrolled at significantly higher rates in more high school mathematics courses (Core-Plus Mathematics, n.d.).

Discussion and Conclusion

Districts are at liberty to group students as they choose, but for districts wanting to open mathematics pathways to more students and create opportunities for greater achievement, this chapter has described many options to consider.

In the elementary years, students should experience common mathematics content that lays a productive groundwork of conceptual understanding for more advanced mathematics. Students work in different ways and at faster or slower rates, but this does not mean that they should be exposed to different content. Many of the teaching approaches and activities described above and in chapters three and six emphasize multidimensional math teaching that supports depth of understanding over speed and memorization. When mathematics questions invite students to engage in reasoning, making connections, and seeing and representing ideas in different ways, they can engage all students appropriately. In addition, strategies like tutoring and personalized supplementary programs can help students secure and reinforce skills that allow them to progress successfully through the curriculum.

Middle schools also have an important role to play in ensuring that all students receive well-taught, challenging coursework that does not close off later options. By maintaining rich mathematical content along with strong and supportive teaching, they give more students access to higher-level mathematics. Given changes in course content with the advent of the CA CCSSM, middle school students can rely on richer algebra content in grade eight, preparing them for Mathematics I or Algebra I courses and more in-depth work with linear functions and exponential functions and relationships. The integrated high school course pathways that start with Mathematics I build directly on the CA CCSSM for eighth grade and provide a seamless transition of content through an integrated curriculum. As noted earlier, schools may also enable students with interest and readiness to begin the high school pathway in middle school. Schools should be mindful of addressing potential curriculum gaps for these students, so that they can be successful. (See chapter eight for discussions of various high school course pathways.)

Even as high schools differentiate mathematics course-taking options, they can open up opportunity for more students to engage in advanced course-taking by reducing the redundancies in current courses that may unnecessarily slow progress toward the highest-level courses like calculus or statistics. This framework proposes that the state convene experts to evaluate options for doing so. For students who do not begin the high school sequence in middle school, high schools can provide multiple ways to reach advanced courses—e.g., through block scheduling or supplementary courses during the school year or summer, by ongoing tutoring opportunities, and/or by offering a range of rigorous third- and fourth-year courses that do not require prior acceleration.

This chapter has described alternative approaches to student grouping, especially in elementary and middle school. In planning course offerings, districts should take note of the California Mathematics Placement Act of 2015, which requires that every high school placement policy of a local educational agency meet the following requirements (CDE, 2016):

- Systematically takes multiple objective academic measures of pupil performance into consideration
- Includes at least one placement checkpoint within the first month of the school year to ensure accurate placement and to permit reevaluation of individual student progress

- Requires an annual examination of pupil placement data to ensure that students are not held back in a disproportionate manner on the basis of their race, ethnicity, gender, or socioeconomic background
- Requires a report on the results of the annual examination by the local educational agency to its governing board or body
- Offers clear and timely recourse for each pupil and his or her parent or legal guardian who questions the student’s placement
- For non-unified school districts, addresses the consistency of placement policies between elementary and high school districts

By designing curriculum and teaching in ways that invite personalization and by providing open-ended tasks wherein many possible approaches enable deep learning, teachers allow more students to tackle ambitious mathematics successfully. Students who are already eager and able mathematicians will be able to excel with a stronger foundation, joined now by more of their peers who gain from greater opportunities to develop their potential.

Long Description for Chapter 9

Figure 9.1: Border Problem

Six rectangles include two squares each. Squares include borders comprised of various shadings. Rectangle one includes two squares shaded to indicate 10 + 10 + 8 + 8 and n + (n – 2) + n + n (n – 2). Rectangle two includes two squares shaded 10 + 9 + 9 + 8 and n + 2(n – 1) + (n – 2). Rectangle three includes two squares shaded 4 × 8 + 4 and 4(n – 2) + 4. Rectangle five includes two squares shaded 9 + 9 + 9 + 9 = 9 × 4 and (n – 1) × 4. Rectangle five includes two squares shaded 4 × 10 – 4 and 4n – 4. Rectangle six includes two squares shaded (10 × 10) – (8 × 8) and n squared – (n – 2) squared.

Return to figure 9.1 graphic

California Department of Education, October 2023