# Mathematics Framework
# Chapter 13: Instructional Materials to Support Equitable and Engaging Learning of the California Common Core State Standards for Mathematics

# Introduction

The California State Board of Education (SBE) has constitutional and statutory authority to adopt instructional materials for kindergarten through grade eight. *Education Code* (*EC*) sections 60200–60204 describe the process for the adoption of instructional materials for these grades and mandate that submitted materials be evaluated for consistency with adopted content standards and specific evaluation criteria approved by

the SBE. The evaluation criteria are updated with each content area adoption to ensure relevancy and are incorporated into the curriculum frameworks.

*EC* Section 60010(h) defines instructional materials as "all materials that are designed for use by pupils and their teachers as a learning resource and help pupils to acquire facts, skills, or opinions or to develop cognitive processes. Instructional materials may be printed or non-printed, and may include textbooks, technology-based materials, other educational materials, and tests." The SBE traditionally adopts only basic instructional materials programs, for example, programs that are designed for use by pupils and their teachers as a principal learning resource and meet in organization and content the basic requirements of a full course of study—generally one school year in length.

Instructional materials that are adopted by the state help teachers to present and students to learn the content set forth in the *California Common Core State Standards for Mathematics with California Additions* (CA CCSSM), which include both the content standards and the standards for mathematical practice (SMPs). This document establishes the criteria for evaluating instructional materials for the current adoption and will help to inform future adoptions. It is the intent of the SBE that these criteria be seen as neutral on the format of instructional materials to ensure consideration of emerging technologies that incorporate digital and interactive online formats, and other innovative types of curriculum.

A local educational agency (LEA) may choose to use instructional materials that have not been adopted by the SBE, pursuant to *EC* Section 60210, so long as they are aligned to state standards and a majority of the participants of any review process conducted by the LEA are classroom teachers who are assigned to the subject area or grade level of the materials being reviewed.

## Intent and Purpose

*Mathematics Framework Chapter 13: Instructional Materials to Support Equitable and Engaging Learning of the CA CCSSM* is intended to support publishers and content developers of instructional materials to serve California's diverse student population.

Those publishers and content developers may choose to participate in the California SBE Instructional Materials Adoption process, and this chapter includes the criteria that will be used for that adoption review and evaluation. In addition, this chapter provides guidance for local districts on the adoption of instructional materials for students in grades nine through twelve, the social content review process, supplemental instructional materials, and accessible instructional materials.

The 10 years since the adoption of California's first CCSS-aligned math framework has been a decade of technological advancements and innovations unimagined in 2013. Changes to instructional materials have been evolving at an equally rapid pace. Educators today deftly shift from paper to screen, book to video, discussion to chat. The global pandemic and the quick shift to remote learning accelerated the use of technology and digital tools to deepen student learning and strengthen student engagement in the classroom. Students today are digital natives—some even the children of digital natives—who mastered the use of tablets and devices even before they could walk. And we cannot imagine now what changes lie in the next 10 years with artificial intelligence heading toward becoming standard in education circles.

Instructional materials for mathematics—the field from which all technology springs—should reflect our twenty-first century world and best practices learned over the last decade of teaching the CA CCSSM. Classroom tools should be dynamic, adapting to our ever-changing world. Evaluations of materials should focus on what evidence shows about how best to teach the standards.

In the face of change, one thing remains constant: high-quality instructional resources help educators teach and students learn. This chapter on instructional materials differs from other chapters of the framework in audience and purpose. The primary audience of this chapter are the publishers and content developers of materials to support mathematics instruction, who will find information they need to participate in the SBE adoption process. A key difference between that guidance and the guidance for teachers and administrators throughout the other chapters of the framework is in addressing content and context. The publisher and content developers of instructional

materials provide the content to address standards, but they should remain aware of the context of the mathematics instruction that will occur using these materials as resources for teachers and students. Bridging the understanding between content and context, and developing instructional resources that provide guidance to teachers while allowing the flexibility necessary for supporting all students, will be critical in the implementation of the 2023 *Mathematics Framework*. For this reason, there is a Publisher Content Developer Guide to the Mathematics Framework section at the end of this chapter.

## Instructional Resources and Focus, Coherence, and Rigor in the Common Core State Standards for Mathematics

Instructional materials for mathematics in California should place a strong emphasis on students' engagement in mathematics in the ways described in the CA CCSSM (or the Standards). Built upon underlying and updated principles of focus, coherence, and rigor, the Standards hold the promise of enabling all California students to become powerful users of mathematics in order to better understand and positively impact the world—in their careers, in college, and in civic life. This promise is best realized when students are actively engaged in questioning, productive struggle, problem solving, reasoning, communicating, and explaining.

For this adoption, publishers and content developers of instructional resources should focus on the mathematical practices and provide guidance to teachers on impactful classroom instruction using the three principles of focus, coherence, and rigor, as embedded in the *Mathematics Framework*.

The principle of focus is closely tied to the goal of depth of understanding. The principle derives from a need to confront the mile-wide but inch-deep mathematics curriculum experienced by many. This framework's answer to the coverage-versus-depth challenge posed by the principle of focus is to lay out principles for instructional design that make the Standards achievable, including: (a) focus on Big Ideas; (b) use tasks worthy of student engagement; and (c) embed exercises in a larger context of investigation.

The challenge posed to curriculum designers by the principle of coherence is to avoid losing the forest for the trees. That is, discrete content standard mastery does not necessarily assemble in students' minds into a coherent big-picture view of mathematics. In other words, students do not arrive at conceptual understanding of mathematical ideas simply by performing procedural tasks. Instructional materials cannot match the contours of the Standards by approaching each individual content standard as a separate event. Nor can materials align to the Standards by approaching each individual grade as a separate event. The Standards are woven out of learning progressions, and maintaining these progressions in the implementation of the Standards is critical to help all students achieve higher level mathematics. This framework's answers to the challenge posed by the principle of coherence are to focus on: (a) Big Ideas; (b) progressions of learning across grades; (c) relevance to students' lives; and (d) high-quality first instruction.

Rigor refers to an integrated way in which conceptual understanding, strategies for problem-solving and computation, and applications are learned, so that each supports the other. The challenge posed by the principle of rigor is to provide all students with experiences that interweave concepts, problem-solving (including appropriate computation), and application, such that each supports the other. It is important the publishers and content developers fully understand the instructional shifts and how their choices of instructional strategies in the materials impacts teachers' and students' ability to access those shifts.

Instructional resources for mathematics include a variety of instructional materials—tools such as rods, cubes, tiles and building materials, rulers, protractors, graph paper, calculators, computers and technology such as online interactive content, interactive whiteboards and student-response devices. The term "instructional materials" is broadly defined to include textbooks, technology-based materials, other educational tools, and assessment instruments.

## State Adoption of Instructional Materials

The California SBE adopts instructional materials for kindergarten through grade eight.

Under current state law, LEAs—school districts, charter schools, and county offices of education—are not required to purchase state-adopted instructional materials. The state-level adoption process determines whether a publisher's or content developer's program has fully addressed all grade-level content standards, as well as the other evaluation criteria, and is not an endorsement of a particular program. For the 2025 mathematics adoption, the Standards are organized around the Big Ideas along the learning progressions, and should be addressed collectively, not individually.

LEAs have the authority and the responsibility to conduct their own evaluation of instructional materials and to adopt the materials that best meet the needs of their students. Additionally, there is no state-level adoption of programs for use by students in grades nine through twelve, however, Algebra I and Integrated Mathematics I (hereafter referred to as Mathematics I) are included in the kindergarten through grade eight adoption process. LEAs have the sole responsibility and authority to adopt additional instructional materials for grades nine through twelve.

The primary source of guidance for the selection of instructional materials is the following section *Criteria for Evaluating Mathematics Instructional Materials for Kindergarten Through Grade Eight* (Criteria). The Criteria section provides a comprehensive description of effective instructional programs that are aligned with the CA CCSSM and are consistent with the guidance in this framework. The Criteria will be the basis for the 2025 Adoption of Mathematics Instructional Materials and is also a useful tool for LEAs that conduct their own evaluations of instructional materials.

## Criteria for Evaluating Mathematics Instructional Materials for Kindergarten Through Grade Eight

Instructional materials that are adopted by the state help teachers to present, and students to learn, the content set forth in the CA CCSSM. This refers to both the content standards and the Standards for Mathematical Practice (SMPs), as revised pursuant to California *Education Code (EC)* Section 60605.11. To accomplish this purpose, this document establishes criteria for evaluating mathematics instructional materials for the

current adoption cycle, which adds greater emphasis to the SMPs. These criteria serve as evaluation guidelines for the statewide adoption of mathematics instructional materials for kindergarten through grade eight.

The Standards require focus, coherence, and rigor as defined above and discussed in more detail in chapter 1 of the *Mathematics Framework*, with development of the content standards and SMPs intertwined throughout. The Standards are organized by grade level in kindergarten through grade eight and by conceptual categories for higher mathematics. The standards for higher mathematics are organized in two ways—as model courses and in conceptual categories. Overall, the Standards do not dictate a singular approach to instructional resources—to the contrary, they provide opportunities to raise student achievement through innovation.

In addition to this Framework, there are a number of supportive and advisory documents that are available for publishers and content developers of instructional materials that define the depth of instruction necessary to support the focus, coherence, and rigor of the CCSSM. These documents include the *Progressions Documents for Common Core Math Standards (*available at https://mathematicalmusings.org/2023/02/28/final-version-of-progressions/) and Smarter Balanced Test Specifications (available at http://www.smarterbalanced.org/).

The Progressions note key connections among standards within and between grades, point out cognitive difficulties and pedagogical solutions, and give more detail on particularly knotty areas of the mathematics. For example, they note connections between kindergarten through grade five Measurement and Data standards and standards for work with numbers (74). They give a side-by-side comparison of the standards for measurement of area in grades three and five and the measurement of volume in grades five and six (89). They display multiplication and division situations for equal groups, arrays, and comparisons (32), and analogous measurement situations (103).

# Three Types of Programs

Three types of programs will be considered for adoption: basic grade-level for kindergarten through grade eight, Algebra I, and Mathematics I. Publishers and content developers may submit programs for one grade or any combination of grades. In addition, publishers and content developers may include intervention and acceleration components to support a range of learners.

## Basic Grade-Level Program

The basic grade-level program is the comprehensive curriculum in mathematics for students in kindergarten through grade eight, or a subset of those grades. Such programs provide the foundation for instruction and are intended to ensure that all students master the CA CCSSM. Publishers and content developers may submit programs for one grade or any combination of grades.

## Algebra I and Mathematics I

The content described in the CA CCSSM for kindergarten through grade eight provides the foundational knowledge for Algebra I or Mathematics I. The course content will be consistent with its high school counterpart and will articulate with the subsequent courses in the sequence. Furthermore, materials for Algebra or Mathematics I might be offered separately or as part of a sequence (e.g., a three-year sequence for middle grade mathematics that uses the CA CCSSM grades six, seven, and eight blueprint, leaving Algebra or Mathematics I as a separate course; or a three-year sequence that incorporates the content for CA CCSSM grades six, seven, and eight with Algebra I or Mathematics I in a more coherent approach).

# Criteria for Materials and Tools Aligned with the Standards

The criteria for the evaluation of mathematics instructional resources for kindergarten through grade eight are organized into five categories:

1. **Mathematics Content/Alignment with the Standards.** CA CCSSM content standards, practice standards, and sequence of the mathematics program provide structure for what students should learn at each grade level.

2. **Program Organization.** Instructional materials support instruction and learning of the Standards, demonstrating how they are grouped around bigger ideas in ways that support coherence and include the instructional guidance features deemed necessary for successful implementation of the program. (These features may include chapter overviews, glossaries, etc.).
3. **Assessment.** A variety of assessment strategies, as defined in chapter 12, are presented in the instructional materials for measuring what students know and are able to do, and guide next steps for teachers.
4. **Access and Equity.** Access to the standards-based curriculum for all students with supports for those with language and learning differences.
5. **Instructional Planning and Support.** Coherent guidelines for teachers to follow when planning to provide effective standards-based instruction and guidance to help teachers provide instruction that ensures opportunities for all students.

Mathematics materials should support teaching to the CA CCSSM as further interpreted through this curriculum framework. To be eligible for adoption, programs must include a well-defined sequence of instructional opportunities that provides a path for all students to become proficient in the standards. While the following are the specified criteria for categories 1–5, the State recognizes that advances in technology, as well as the multiple pathways for student proficiency in the Standards, allow for production of mathematics materials in many different forms that will support instruction and learning of mathematics that will meet the criteria set forth below.

Materials that fail to meet all of the criteria in category 1 (Mathematics Content/Alignment with the Standards) will not be considered suitable for adoption. The criteria for category 1 must be met in the core materials or via the primary means of instruction, rather than in ancillary components. In addition, programs must have strengths in each of categories 2 through 5 to be suitable for adoption.

## Category 1: Mathematics Content/Alignment with the Standards

Mathematics materials should support teaching to the CA CCSSM as further interpreted through this curriculum framework. To be eligible for adoption, programs must include a well-defined sequence of instructional opportunities that provides a path for all students to become proficient in the standards.

All programs must include the following features:

1. Instructional materials, as defined in *EC* Section 60010(h), must be aligned to the CA CCSSM Content Standards and SMPs, adopted by the SBE in August 2010 and modified in January 2013.
2. Instructional materials must be consistent with the content of the 2023 *Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve* (*Mathematics Framework*), and the depth of understanding of mathematics and mathematics instruction as described in the Publishers' and Content Developers' Guide to the *Mathematics Framework* section in this chapter. Materials develop conceptual understanding of key mathematical concepts and offer engaging applications of the mathematics, using real-world examples and data as a means to spark inquiry and apply mathematical concepts.
3. Instructional materials shall be accurate and use proper grammar and spelling (*EC* Section 60045).
4. Instructional materials include instructional content based on the California Environmental Principles and Concepts developed by the California Environmental Protection Agency and adopted by the SBE (*Public Resources Code* Section 71301) where practicable and aligned to the guidance in the *Mathematics Framework.*

## Category 2: Program Organization

The organization and features of the instructional materials support instruction and learning of mathematics. Instructional materials must have strengths in these areas to be considered suitable for adoption:

1. The instructional materials are consistent with the progressions in the Standards and guidance in this curriculum framework for relating content to the concepts of the Big Ideas in previous and future grades, and fully integrate content into strategically designed opportunities for students to use the mathematical practices. Further information regarding the Big Ideas of mathematics may be found in the Publishers' and Content Developers' Guide to the *Mathematics Framework* Section in this chapter.
2. In each grade in the kindergarten through grade eight sequence, the instructional materials are designed for students and teachers to spend the large majority of their time on mathematical investigations that address the Big Ideas of that grade, as described above, and in the grade band chapters of the *Mathematics Framework*.
3. Materials drawn from other subject-matter areas are consistent with the currently adopted California standards at the appropriate grade level, including the *California Career Technical Education Model Curriculum Standards* where applicable.
4. Intervention components, if included, are designed to help teachers respond to students' progress in mathematics, with opportunities to reclaim missed concepts from prior grades, to give growth mindset messages and communicate that all students can be successful and to give students access to rich, connected ideas, helping them to develop number flexibility as defined in the *Mathematics Framework*.
5. Instructional materials include supporting activities that provide students opportunities to access grade-level mathematics and reason mathematically in age-appropriate contexts, with scaffolds that provide needed foundations or expand depth to provide additional challenges targeted to deeper understanding.

6. Teacher and student materials contain an overview of the chapters or units, clearly identify the target mathematical concepts and practices, and include clear organizers. These may include tables of contents, indexes, glossaries that clarify important mathematical terms, and/or their technology-based resource equivalents.
7. The grade-level standards, Big Ideas, and the SMPs shall be explicitly stated in the student editions demonstrating alignment with student lessons.
8. The instructional materials shall include content, including assessments and all instruction-related activities, for the equivalent of instruction to address a full school year in each grade.
9. A list of the CA CCSSM, organized around and within the major concepts, is included in the teacher guidance, together with page-number citations or other references that demonstrate alignment with the content standards and SMPs.

## Category 3: Assessment

Instructional materials should contain strategies and tools for continually assessing student understanding and opportunities for new learning. Instructional materials in mathematics must have strengths in these areas to be considered suitable for adoption:

1. Student and teacher materials include formative assessments to provide multiple methods to assess student understanding to inform instruction, such as graphic organizers, student observation, student interviews, journals and learning logs, mathematics portfolios, self- and peer evaluations, tests and quizzes, self-reflection, and performance tasks.
2. Student and teacher materials include summative assessments to provide multiple methods of assessing what students have learned and are able to do, such as selected response, constructed response, real-world problems, performance tasks, rubrics, and open-ended questions.
3. Assessments integrate mathematics content and the language needed to participate in the Standards for Mathematical Practice.

4. Teacher materials include suggestions on the use of assessment data to guide decisions about instructional practices, and on ways to modify instruction so that all students are consistently progressing toward meeting or exceeding the standards.
5. At each grade level, instructional materials provide assessment practices (e.g., entry-level, diagnostic, formative, interim, skill-based, and summative) necessary to prepare all students for success in higher mathematics instruction.
6. Teacher and student materials include curriculum-embedded assessments that permit teachers to scaffold student learning. Teacher materials should also provide guidance for diagnostic feedback.

## Category 4: Access and Equity

Resources should incorporate recognized principles, concepts, and research-based strategies to meet the needs of all students and provide equal access to learning through lessons that are relevant to the students. Instructional resources should include suggestions for teachers on how to differentiate instruction to meet the needs of all students. In particular, instructional resources should provide guidance to support students who are English learners, at-promise, advanced learners, and students with learning disabilities. Instructional resources must have strengths in these areas to be considered for adoption:

1. Instructional materials include resources for specific student populations that would benefit from supports such as, but not limited to, culturally responsive materials for English learner and other linguistically and culturally diverse students; strategies that reflect Universal Designs for Learning; and scaffolds that allow for work along the learning progressions in response to student needs.
2. Student materials are appropriate for use with a wide range of learners.
3. Teacher materials include comprehensive teacher guidance and differentiation strategies that are tied to the *Mathematics Framework*, based on current and confirmed research, to adapt the curriculum to meet students' identified special needs and to provide effective, efficient instruction for all students.

4. Teacher materials include strategies for students who are English learners that are consistent with the *California English Language Development Standards: Kindergarten Through Grade 12* adopted under *EC* Section 60811. In addition, the resource Improving Education for Multilingual and English Learner Students: Research to Practice contains a wealth of guidance, resources, and tools for helping schools better meet the needs of multilingual and English learner students (CDE, 2020).
5. Teacher materials include strategies to help students who have not yet achieved grade level proficiency in reading, writing, speaking, and listening in academic English to understand the mathematics content and practices that are tied to the *Mathematics Framework*.
6. Suggestions for advanced learners that are tied to the *Mathematics Framework* and that allow students to study grade-level content in greater depth.
7. The visual design of the materials does not distract from the mathematics, but instead serves to support students in engaging thoughtfully with the subject.

## Category 5: Instructional Planning and Support

Instructional materials must contain a clear road map to assist teachers when planning instruction for the specific needs and context of their students. The instructional resources should support Universal Design for Learning (UDL) and culturally and linguistically responsive instruction to improve and optimize teaching and make learning more equitable for all people based on scientific insights into how humans learn. Instructional materials in mathematics should have strengths in many of these areas to be considered suitable for adoption:

1. A teacher's edition that explains the role of the grade-level mathematics concepts in the context of the overall mathematics curriculum for kindergarten through grade twelve.
2. Materials provide teacher guidance that includes annotations and suggestions for how to utilize and implement the student and ancillary materials, with specific attention to engaging students to guide their mathematical development.

3. Unit and/or lesson plans, including suggestions for organizing resources in the classroom and ideas for pacing or scope and sequence of instruction.
4. A curriculum guide for the academic instructional year.
5. Answer keys for any workbooks, quizzes, or other related student activities, where appropriate.
6. Materials make use of concrete representations, including manipulatives, audiovisual, multimedia, and interactive technology resources that support instruction of the CA CCSSM, and include clear instructions in their use for teachers and students. Where materials integrate technology – such as interactive tools, virtual manipulatives/objects, and / or dynamic mathematics software – they do so in ways that engage students in applying the standards.
7. Optional homework activities, if included, should extend and reinforce classroom instruction and provide additional practice of mathematical content, practices, and applications that have been taught.
8. Materials provide examples of student work and representation of possible student strategies to orient teachers to student thinking and help teachers elicit, make sense of, and respond to student thinking.
9. Specific strategies to support students in developing the language skills needed to meet the mathematical learning and language objectives that are explicitly and clearly associated with instruction and assessment.
10. Teacher guidance that contains explanations and examples of mathematics concepts.

## Guidance for Instructional Materials for Grades Nine through Twelve

The Criteria document (above) is intended to guide publishers and content developers in the development of instructional materials for students in kindergarten through grade eight. It also provides guidance for selection of instructional materials for students in grades nine through twelve. The five categories in the Criteria document are an appropriate lens through which to view any instructional materials a district or school is considering purchasing. Additional guidance for evaluating instructional materials for

grades nine through twelve is provided in the *High School Publishers' Criteria for the Common Core State Standards for Mathematics* (NGA/CCSSO, 2013).

The process of selecting instructional materials at the district or school level usually begins with the appointment of a committee of educators, including teachers and curriculum specialists, and possibly students, who determine what instructional materials are needed, develop evaluation criteria and rubrics for reviewing materials, and establish a review process that involves teachers and content-area experts on review committees. After the review committee develops a list of instructional materials that are being considered for adoption, the next step is to pilot the instructional materials. An effective piloting process helps determine if the materials provide teachers with the resources necessary to implement an instructional program based on the CA CCSSM. One resource on piloting is the SBE policy document "Guidelines for Piloting Textbooks and Instructional Materials," which is available through the California Department of Education (CDE) (CDE, 2015).

Selection of instructional materials at the local level is a time-consuming but very important process. Poor instructional materials that are not fully aligned with the principles of focus, coherence, and rigor as defined in the 2023 *Mathematics Framework* and the CA CCSSM waste precious instructional time. High-quality instructional materials support effective instruction and student learning of concepts, mathematical practices, and language needed to express them.

## Social Content Review

To ensure that instructional materials reflect California's diverse society, avoid stereotyping, and contribute to a positive learning environment, instructional materials used in California public schools must comply with the state laws and regulations that involve social content. Instructional materials must conform to *Education Code* sections 60040–60045, as well as the SBE's *Standards for Evaluating Instructional Materials for Social Content* (CDE, 2013). Instructional materials that are adopted by the SBE meet the social content requirements. The CDE conducts social content reviews of a range of instructional materials and maintains a searchable database of the materials that meet these social content requirements (CDE, n.d.a).

If an LEA intends to purchase instructional materials that have not been adopted by the state or are not included on the list of instructional materials that meet the social content requirements maintained by the CDE, then the LEA must complete its own social content review. Information about the review process is posted on the CDE's Social Content Review web page (CDE, 2013).

## Accessible Instructional Materials

The CDE's Clearinghouse for Specialized Media and Technology (CSMT) provides instructional resources in accessible and meaningful formats to students with learning differences and identified disabilities, including students who have hearing or vision impairments, severe orthopedic impairments, or other print disabilities. The CSMT produces accessible versions of textbooks, workbooks, literature books, and assessment books. Specialized instructional materials include braille, large print, audio recordings, digital talking books, electronic files, and American Sign Language video books. Local assistance funds finance the conversion and production of these specialized materials. The distribution of various specialized media to public schools provides general education curricula to students with disabilities. Information about accessible instructional materials and other resources, including what is available and how to order, is posted on the CSMT's Media Ordering Guide page (CDE, n.d.b).

# Publishers' and Content Developers' Guide to the Mathematics Framework

To address the needs of California educators in 2023, the *Mathematics Framework* includes several new emphases and types of chapters. Instead of two separate chapters, one on instruction and one on access, a single chapter, *Chapter Two: Teaching for Equity and Engagement*, promotes instruction that fosters equitable learning experiences for all children, and challenges the deeply-entrenched policies and practices that lead to inequitable outcomes. Good teaching leads to equitable and higher outcomes. Instruction and equity come together to create instructional designs that bring about equitable outcomes. The commitment to equity extends throughout the framework and every chapter considers the ways in which equity may be brought about. Publishers and content developers should consider the lens of equity as discussed in the *Mathematics Framework* when developing lessons and units for instructional materials.

Students at all levels learn best when they are actively engaged in questioning, struggling, problem solving, reasoning, communicating, and explaining. Powerful mathematics classrooms require students to have a sense of agency (a willingness to engage in the discipline, based in a belief in progress through engagement) and an understanding that the intellectual authority in mathematics rests in mathematical reasoning itself (in other words, that mathematics makes sense) These factors support students' development of their own identities as powerful math learners and users. Further, active-learning experiences enable students to engage in a full range of mathematical activity—exploring, noticing, questioning, solving, justifying, explaining—making clear that mathematics is far more than calculating. Homework activities allow students to reflect on the concepts learned that day. Publishers and content developers should consider this research when developing activities for lessons and units.

Three concepts of instructional resources that will be critical for publishers and content developers as they develop materials are content coverage, content depth, and content delivery. Content coverage refers to alignment to the mathematics standards, including the SMPs. Content depth refers to the ability of the materials to be used by teachers to

provide instruction for a deep understanding of the mathematical practices and application of mathematics, focusing on the Big Ideas and learning progressions. Content delivery refers to the guidance to teachers on how to provide high-quality mathematics instruction within the specific instructional pedagogy, scope and sequence of the materials.

The *Mathematics Framework* addresses the challenge posed by the principle of coherence through the shifts of Big Ideas, progressions across grades (thus, grade-band chapters rather than individual grade chapters), and relevance to students' lives. A big idea is characterized by including connected mathematical content and a driver for investigation—*it is the combination of content and investigation that makes content meaningful and important*.

The four content connections described in the framework organize content and provide mathematical coherence through the grades:

- CC1 Reasoning with Data
- CC2 Exploring Changing Quantities
- CC3 Taking Wholes Apart, Putting Parts Together
- CC4 Discovering Shape and Space

These content connections should be developed through investigation of questions in authentic contexts; these investigations will naturally fall into one or more of these Drivers of Investigation:

- DI1: Making Sense of the World (Understand and Explain)
- DI2: Predicting What Could Happen (Predict)
- DI3: Impacting the Future (Affect)

Big ideas that drive design of instructional activities will link one or more content connections with a driver of investigation, such as Communicating Stories with Data to Predict What Could Happen, or Exploring Changing Quantities to Impact the Future. Instructional materials should primarily involve tasks that invite students to make sense of these Big Ideas, elicit wondering in authentic contexts, and necessitate mathematics.

Big ideas in math are central to the learning of mathematics, link numerous mathematical understandings into a coherent whole, and provide focal points for students' investigations. An authentic activity or problem is one in which students investigate or struggle with situations or questions about which they actually wonder. Lesson design should be built to elicit that wondering. An activity or task necessitates a mathematical idea or strategy if the attempt to understand the situation or task creates for students a need to learn or use the mathematical idea or strategy.

Publishers and content developers should consider UDL when developing lessons and activities in their materials. It is critical for publishers and content developers to understand that UDL is a framework for instructional planning for all students and not an intervention strategy to be employed for special populations.

Any intervention strategies included in the instructional program should be aligned to the CA CCSSM.

Publishers and content developers should consider the following terms and their application to mathematics when developing instructional materials:

Big Idea: Big ideas in math are central to the learning of mathematics, link numerous math understandings into a coherent whole, and provide focal points for students' investigations. So a focused set of big ideas, indicated as Big Ideas, was created as part of the California Digital Learning Integration and Standards Guidance initiative (CDE, 2021). These grade level Big Ideas, organized by Content Connections, and inclusive of multiple CA CCSSM content standards, are presented in the grade-banded chapters, 6, 7, and 8.

Authentic: An authentic context, activity, or problem is one in which students investigate or struggle with situations or questions about which they actually wonder. Lesson design should be built to elicit that wondering. In contrast, an activity is inauthentic if students recognize it as a straightforward practice of recently-learned techniques or procedures, including the repackaging of standard exercises in forced "real-world" contexts. Mathematical patterns and puzzles can be more authentic than such "real-

world" settings.

Necessitate: An activity or task necessitates a mathematical idea or strategy if the attempt to understand the situation or task creates for students a need to understand or use the mathematical idea or strategy.

Instructional Practice: The shifts in the *Mathematics Framework*, and subsequent professional learning opportunities for implementation, will focus on the instructional practices of teachers. Many teachers have experienced mathematics as a set of procedures to be memorized, so it is critical that they receive opportunities to experience mathematics differently themselves. When teachers work on rich mathematics tasks, through which they can ask their own questions, reason and communicate with others, develop curiosity and wonder, they start to see mathematical connections that they may never have seen before. This often prompts teachers to change their relationship with mathematics, which is an important precursor to changing their teaching.

Integrated: The type of integration outlined here (implementing the content standards laid out in the CA CCSSM) emphasizes both aspects of integration described in chapter 2: opportunities for forming connections between mathematics and students' experiences, and opportunities to connect different mathematical ideas. In keeping with the thrust of this framework, curriculum and instruction should take both of these into account. As further motivation for integration, NCTM has called for classroom instruction to rely upon reasoning and sense making in an integral way, every day (NCTM, 2009). In order for students to engage in reasoning and sense-making about mathematics, explicit attention to the language needed to do so must be built into the teacher and student materials (see Moschkovich, 2012). Since mathematical competence has been shown to be dependent upon reasoning and sense-making (National Research Council, 2001), curriculum is needed that provides rich opportunities for students to practice reasoning and sense-making in authentic situations.

The *Mathematics Framework*, chapter 4, focuses on key ideas that bring the SMPs to life. The focus is on three interrelated practices: constructing viable arguments and

critiquing the reasoning of others; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. By considering these practices together when developing resources, instructional materials can offer the foundations of classroom experiences that center exploring, discovering, and reasoning with and about mathematics. This vision for teaching and learning mathematics comes out of a several decades-long national push in mathematics education to pay more attention to supporting kindergarten through grade twelve students in becoming powerful users of mathematics to help make sense of their world. Throughout the chapter, the framework explores the practices across the elementary, middle, and high school grade bands. The framework emphasizes students' progression in socializing into the mathematical practices, including some ways in which contexts for learning and doing mathematics and the practices themselves might evolve over the grades.

Across the grades, students use everyday contexts and examples in order to explore, discover, and reason with and about mathematics. At the early grades, everyday contexts might come from familiar activities that children engage in at home, at school and within their community. These contexts might include imagined play or familiar celebrations with friends, siblings, or cousins; and familiar places such as a park, playground, zoo, or school itself. As teachers get to know their students and their students' communities, the contexts that matter to young children come to the fore.

In the middle grades, the contexts that are relevant to students continue to include, but increasingly go beyond, local everyday activities and interactions. Middle-school students might begin to explore publicly available datasets on current events of interest, use familiar digital tools to explore the mathematics around them, and explore mathematical topics within everyday contexts like purchasing snacks with friends, playing or watching sports, or saving money. By high school, students have available a wide array of contexts to explore, increasingly understanding society and the world around them through explorations in data, number, and space.

As noted in the CA CCSSM, the SMPs remain the same across the entirety of kindergarten through grade twelve. They develop in relation to progressions in

mathematics content. At the elementary level, students work with numbers with which they are currently familiar, and begin to explore the structure of place value, patterns in our base-ten number system (such as even and odd numbers), and mathematical relationships (such as different ways to decompose numbers or relationships between addition and multiplication). Through these explorations, young students conjecture, explain, express agreement and disagreement, and come to make sense of data, number, and shapes.

Students in middle school build on these early experiences to deepen their interactions with mathematics and with others as they do mathematics together. During the elementary grades, students typically draw on contexts and on concrete manipulatives and representations in order to engage in mathematical reasoning and argumentation. At the middle school level, students continue to reason with such concrete referents, and also begin to draw on symbolic representations (such as expressions and equations), graphs, and other representations which have become familiar enough that students experience them as concrete. Middle-school students deepen their opportunities for sense-making as they move into ratios and proportional relationships, expressions and equations, geometric reasoning, and data.

By high school, students continue to build on earlier experiences as they make sense of functions and ways of representing functions, relationships between geometric objects and their parts, and data arising in contexts of interest. As students build on years of making sense of and communicating about mathematics with one another and the teacher, the same practices that cut across transitional kindergarten through grade twelve emerge at developmentally and mathematically appropriate levels.

## Conclusion

Instructional materials that are adopted by California help teachers to present and students to learn the content and practices set forth in the CA CCSSM. As publishers develop these materials, the three critical concepts to keep in mind are content coverage, content depth, and content delivery. In keeping with this framework, materials should strongly emphasize student engagement and provide the foundation for

classroom experiences that center exploring, discovering, and reasoning with and about mathematics. Materials should also provide guidance to teachers on impactful classroom instruction, using the principles of focus, coherence, and rigor, as embedded in this framework.

This chapter has spelled out the criteria the state will use when evaluating materials for adoption. Of particular importance is attending to this framework's lens of equity when developing lessons and units to serve California's diverse student population.

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