Mathematics Framework
Chapter 10: Supporting Educators in Offering Equitable and Engaging Mathematics Instruction

Introduction

As chapters 2 and 9 discuss, teaching for equity and engagement is as rewarding as it is complex. Teachers who use the big ideas approach and teach mathematics by way of carefully designed, intriguing investigations see their students come alive through exploration and discovery; students see what math can do and are motivated to go deeper as they experience their own math capability. However, since most teachers did not learn math this way, they need support to rethink math teaching and acquire skills and strategies that result in the changes in practice vital to improving student learning. This chapter is about how to ensure such teacher support by planning and designing a broad system of structured, ongoing, professional learning programs.

A System of Professional Learning and Support for Mathematics Teachers

As students learn and process mathematics, their teachers learn the effects of their teaching practices and make refinements. These complementary processes form the core learning environment for mathematics. As detailed in earlier chapters, student success depends on enabling teachers to create a learning environment that is equitable and engaging. That requires providing teachers with a broad system of ongoing professional learning and support. Administrators and teacher leaders, such as coaches and teachers on special assignment, provide the initial, programmatic layers of support, while parents, counselors, and community members co-create an interconnected system that supports children and adolescents as they learn.

How can leaders design systems that effectively provide needed professional learning and teacher support?[1] First, designers need clarity on what is meant by professional learning. In this framework, professional learning refers to planned and organized processes that actively engage educators in cycles of continuous improvement guided by the use of data and active inquiry around authentic problems and instructional practices (Coggshall, 2012; Darling-Hammond, Hyler, and Gardner, 2017). Within that definition, those planning mathematics professional learning—including administrators and teacher leaders at the local, state, and county levels—need to be grounded in key priorities that underlie an effective system's design. That is, they need to understand and embrace the vision for mathematics teaching and learning, the major strands of mathematics practices and content as teaching progresses through the grades, and the primacy of equity.

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[1] To provide consistency across subjects for those creating professional learning opportunities, this chapter mirrors chapter 12 (Implementing High-Quality Science Instruction: Professional Learning, Leadership, and Supports) of the *California Science Framework* (CDE, 2016) and echoes many of its recommendations for supporting quality instruction.

The vision for mathematics teaching and learning. This framework embodies the vision that guides creation of effective professional learning programs for mathematics teaching. As described in chapter 2, the goal of mathematics teaching and learning is "for students to view mathematics as a vibrant, inter-connected, beautiful, relevant, and creative set of ideas." Chapter 2 details five components of equitable and engaging teaching for all students that nurture this view of mathematics:

1. Plan teaching around big ideas
2. Use open, engaging tasks
3. Teach toward justice
4. Invite student questions and conjectures
5. Center reasoning and justification

In addition, Darling (2019) provides a framework that is important for supporting linguistically and culturally diverse English learners as well as other students:

1. Take an asset approach and recognize multilingualism as a power
2. Include group work (strategically grouping for language development)
3. Make work visual (include graphic organizers and visual examples and encourage visual communication)
4. Build on students' lived experiences and cultures (allow native language use)
5. Scaffold learning and language development (including sentence frames and sentence starters)
6. Give opportunities for pre-learning (giving students opportunities to learn some prerequisite material ahead of time)

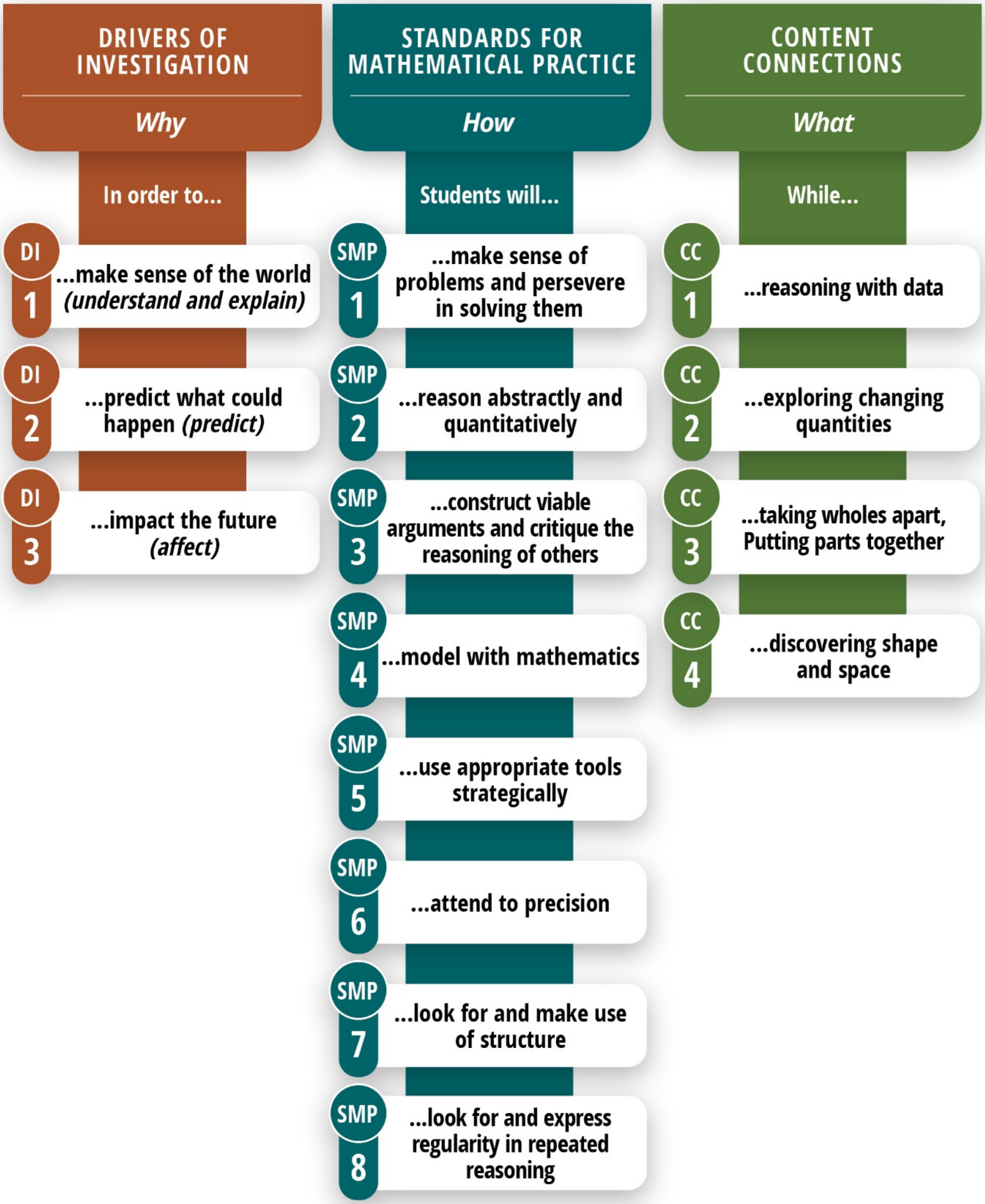
Professional learning experiences for teachers, teacher leaders, and administrators must be designed to support instruction that implements these themes.

***The major strands of mathematics practices and content that progress through the grades**.* Chapters 3 through 5 of this framework illustrate how instruction progresses across the transitional kindergarten through grade twelve continuum through the development of major mathematical strands—that is, mathematical practices and content. Chapters 6 through 8, the grade-band chapters, further detail ways educators can maintain a focus on big ideas and implement instruction in developmentally appropriate ways.

Big ideas are central to the learning of mathematics, link numerous mathematics understandings into a coherent whole, and provide focal points for student investigations (Charles, 2005)—i.e., authentic activities or projects that are the backbone of teaching the big ideas. An authentic activity or problem is one in which students investigate or struggle with situations or questions about which they actually wonder. Lesson design should be built to elicit that wondering. In contrast, an activity is inauthentic if students recognize it as straightforward practice of recently learned techniques or procedures, including the repackaging of standard exercises in forced “real-world” contexts. Mathematical patterns and puzzles can be more authentic than such “real-world” settings.

Throughout the grades, classroom investigations or activities, designed around big ideas, are framed by a conception of the why, how, and what of mathematics—a conception that makes connections across different aspects of content and also connects content with mathematical practices. Three Drivers of Investigation (DIs)—sense-making, predicting, and having an impact—provide the “why” of an activity. Eight Standards for Mathematical Practice (SMPs) provide the “how.” And four types of Content Connections (CCs)—which ensure coherence throughout the grades—provide the “what.” Figure 10.1 maps out the interplay at work when this conception is used to structure and guide student investigations. Because instruction is tied to these three dimensions, this instructional design approach should play a major role in the design of professional learning.

Figure 10.1 The *Why*, *How*, and *What* of Learning Mathematics



Note: *The activities in each column can be combined with any of the activities in the other columns.*

[Long description of figure 10.1]

The primacy of equity. Mathematics education has a long history of inequitable access to rich learning (see chapters 1, 2, and 9 for more discussion of this topic). It is incumbent on all in education, at state, county, district, site, and departmental levels, to work together to create, adapt, and implement professional learning experiences designed to help teachers challenge and overcome the legacy practices that continue to perpetuate these inequities in access and attainment. Even when professional learning is designed with a different primary focus (mathematical practices, particular instructional routines, or teaching big ideas, for instance), its implementation should be relevant to students' cultural backgrounds and existing funds of knowledge. It should also include awareness of and attention to the impacts of unconscious bias on students' experiences in the mathematics classroom.

Professional learning opportunities should highlight equity alongside focus on content and motivation; each of these plays an important role in promoting improved outcomes in math classes. Equity cannot be an afterthought to more traditional content-centered offerings that do nothing to address the fact that "Black, Latinx, Indigenous, women, and poor students, have experienced long histories of underrepresentation in mathematics and mathematics-related domains" (Martin, 2019; see also Martin, Anderson, and Shah, 2017). Inequities caused by systemic issues have resulted in a "culture of exclusion" that persists even in equity-oriented teaching (Louie, 2017). Students' perceptions of their capacity to succeed in mathematics are shaped by messaging from teachers and society. Many efforts in recent years have focused on increasing rates of success among members of historically underrepresented groups in mathematical fields. These include expanded professional training in effective pedagogical practices as well as greater attention on role models and kinds of materials used in the classroom.

It is important for educators to be provided with explicit connections, references, and links to descriptions and supports for the implementation of English learner–centered strategies such *as sentence frames, leveled prompts, vocabulary banks, cognate study, intentional groupings,* and *the use of primary language as support,* among others. These provide purposeful experiences for English learners to engage with language and mathematical concept development as they deepen their knowledge of the SMPs. In addition to the resources listed below, several vignettes in this framework, especially in chapters 2, 6, and 7, include specific guidance to help teachers understand and implement instruction that supports English learners. Moreover, in the transition to increased hybrid learning, accommodations and connections to the California English Language Development Standards (ELD Standards) and online resources should be explicitly addressed, applied, and incorporated into in-person and virtual asynchronous and synchronous lessons. Teachers need to continue using online multilingual resources as well as online platforms to help communicate expectations for both students and parents.

Critical Content for Professional Learning

Due to the inherent complexity of teaching, there is a risk of trying to do everything at once. But for programs to be effective, it is important to design opportunities around a manageable subset of critical content areas. Figure 10.2 (adapted from the 2014 *English Language Arts/English Language Development Framework [ELA/ELD Framework]*) outlines major content areas from which designers of professional learning programs can draw.

Figure 10.2 Critical Content for Professional Learning in Mathematics Education

Establishing a Vision for California's Students

- Develop the readiness for college, careers, and civic life
- Attain the capacities of numerate individuals
- Become broadly literate in quantitative subjects
- Acquire the skills for living and learning in the twenty-first century

Understanding the Standards

- California Common Core State Standards for Mathematics (CA CCSSM) Mathematical Practice Standards
- CA CCSSM Content Standards
- ELA and ELD Standards as implemented in mathematics classes
- Implementing science, history/social studies, career and technical education, and

other standards in tandem with mathematics

Establishing the Context for Learning

- Integrating the curricula
- Motivating and engaging learners
- Teaching from big ideas, not individual standards
- Respecting learners and the cultural and linguistic assets they bring
- Ensuring intellectual challenge

Enacting the Key Themes of Mathematics Instruction

- Mathematics as tools for solving authentic problems in authentic contexts
- Meaning making
- Mathematical practices
- Language development
- Effective expression
- Content knowledge

Addressing the Needs of Diverse Learners

- Comprehensive English language development: integrated and designated ELD
- Additive approaches to language and mathematics development
- Meeting the needs of students with disabilities and students experiencing difficulty
- Meeting the needs of advanced learners and other populations

Exploring Approaches to Teaching and Learning

- Teaching through investigation
- Models of instruction
- Culturally and linguistically responsive teaching
- Supporting biliteracy and multilingualism
- Supporting students strategically (including Universal Design for Learning [UDL] and the Multi-Tiered System of Support [MTSS])

Sharing the Responsibility

- Collaborating within and across grades, departments, and disciplines
- Promoting teacher leadership
- Partnering with community groups and higher education
- Collaborating with parents

Evaluating Teaching and Learning

- Types and methods of assessment (formative, summative, rubrics, portfolios, diagnostic)
- Cycles of assessment (short, medium, long)
- Student involvement in assessment
- Appropriate preparation for state assessments

Integrating Twenty-First Century Learning

- Critical thinking skills
- Creativity and innovation skills
- Communication and collaboration skills
- Community awareness leading to global awareness and competence
- Technology skills

Source: Adapted from the 2014 *ELA/ELD Framework*

Professional Learning Throughout a Teacher's Career

As noted above, in this framework professional learning refers to planned and organized processes that actively engage educators in cycles of continuous improvement guided by the use of data and active inquiry around authentic problems and instructional practices (Coggshall 2012; Darling-Hammond, Hyler, and Gardner, 2017). Teachers' learning occurs in many contexts, including by way of working with students in the classroom, interacting with peers, communicating with administrators, attending conferences, enrolling in online courses, and reading publications. This section describes

- important aspects of professional learning at different stages of an educator's career, with particular focus on characteristics of effective professional learning;

- considerations for planning effective professional learning at each career stage; and
- discussion of various models and strategies for professional learning, with several vignettes illustrating the models and how they incorporate characteristics of effective professional learning.

Understanding key shifts in thinking about professional learning will help designers develop programs that effectively improve teaching practice. Figure 10.3, adapted from the National Comprehensive Center for Teacher Quality's publication *Toward the Effective Teaching of New College- and Career-Ready Standards: Making Professional Learning Systemic* (Coggshall, 2012), summarizes those shifts.

Figure 10.3 Key Shifts in Thinking About Professional Learning

Moving From	Moving Toward
Believing that professional development is some people's responsibility	Believing that professional learning focused on student learning outcomes is everyone's job
Thinking individual goals for professional development are separate from school site and district goals	Aligning individual goals with school site and district goals to provide greater coherence
Using professional development as a means of addressing deficiencies	Embedding professional learning in continuous improvement
Seldom addressing standards for professional learning	Using standards for professional learning
Providing professional development that takes place outside of school, away from students, and is loosely connected to classroom practice	Embedding professional learning in the daily work of teaching so that staff can learn collaboratively and can support one another as they address real problems and instructional practices of their classrooms
Engaging staff in professional development unrelated to data and the continuous improvement process	Engaging staff in a cycle of continuous improvement, guided by the use of active inquiry and multiple sources of evidence

Moving From	Moving Toward
Providing one-shot or short-term professional development with little or no transfer to the classroom	Sustaining continuous professional learning through follow-up, feedback, and reflection to support implementation in the classroom
Limiting professional development based on scarce resources and discrete funding sources	Dedicating and reallocating resources to support professional learning as an essential investment

Source*:* Coggshall, 2012.

Professional learning occurs across all key stages of a teacher's career—preparation, induction, and in-service—as follows.

Teacher Preparation

Since CA CCSSM-aligned instruction is different in significant ways from the school mathematics experience of most teachers, the phases of new teacher preparation and induction are key factors in providing a pipeline of teachers with the skills and knowledge to provide high-quality CA CCSSM-aligned instruction. Educators of pre-service teachers need to align their programs to reflect the authentic-context, big-idea-based instruction described in this framework so that pre-service teachers have the opportunity to experience it as learners. Factors to consider in the development of CA CCSSM-aligned teacher preparation programs include the following:

- Early field experience hours that are dedicated to observing and interacting with students and teachers in authentic mathematics classroom environments.
- Student teaching opportunities that include content-rich experiences and integrated learning experiences.
- Mathematics and mathematics methods classes that address mathematics as a collection of tools and lenses for making sense of authentic contexts, with emphasis on learning mathematical ideas through the mathematical practices and active-learning pedagogy rather than passive lecture.

- Mathematics and mathematics methods classes that develop mathematics through asset-based, culturally and linguistically relevant and sustaining pedagogy.
- Mathematics methods classes that address pedagogical content knowledge that facilitates student conceptual understanding of content standards over time and how to address incorrect, developing, and alternative student conceptions of those ideas.
- Student teaching experiences with mathematics teachers who are effectively incorporating CA CCSSM.
- Effective examples of the development of mathematical ideas through the investigation of authentic contexts and problems (in both pre-service teacher course work and student teaching).
- Mathematics methods classes that address how to organize instruction around big ideas and meaningful investigations, rather than isolated standards.
- Mathematics and mathematics methods classes that explore mathematics, and the teaching and learning of mathematics, from many cultures. By taking the time to acknowledge and center contributions to mathematical understanding from Africa, South America, Asia, and indigenous peoples around the world, educators can ensure that students can better appreciate the global nature of mathematical discovery. In a similar way, prospective teachers in methods courses can expand their understanding of teaching and learning mathematics by exploring a variety of approaches from a diverse array of cultures. Mathematics methods classes can make evident ways in which language and content are interconnected and mutually reinforcing; one cannot develop without the other. Language needed for disciplinary thinking and concepts should not be taught in isolation but in the context of what students relate to and need to know to access and communicate mathematical thinking. Opportunities to practice language and communicate understanding must be integrated (e.g., students have the opportunity to gain ideas from a discussion or a reading before writing).

Additionally, mathematics education faculty and other educators (e.g., university field advisors, master cooperating teachers) who provide pre-service instruction must be grounded in the CA CCSSM-relevant knowledge and skills to facilitate their students' (pre-service teachers) ability to address the CA CCSSM's vision. Important resources for guiding the design of high-quality teacher preparation programs and, specifically, mathematics teacher preparation programs include the Learning Policy Institute's *Effective Teacher Professional Development* (Darling-Hammond, Hyler, and Gardner, 2017); *Preparing Teachers—Building Evidence for Sound Policy* (NRC, 2010); *Powerful Teacher Education, Lessons from Exemplary Programs* (Darling-Hammond, 2006); the National Council of Teachers of Mathematics' Professional Development Guides (NCTM, n.d.); and *Mathematical Education of Teachers II, Conference Board of the Mathematical Sciences* (Conference Board of the Mathematical Sciences [CBMS], 2012).

Induction for New Teachers

Teaching is hard and thoughtful work. It is not uncommon for new teachers to feel isolated and burdened by the demands (both managerial and instructional) of preparing for and working in a classroom. The implementation of effective preparation and support programs specifically tailored to the needs of new teachers can alleviate these issues to a large degree. Induction program designers should consider doing the following to provide support for prospective teachers of mathematics:

- Redefine the professional dynamics of the teacher induction process by pairing beginning mathematics teachers with experienced mathematics teachers who can act as mentors rather than delegators. This connection may help address the need for inclusion and community and may provide new teachers with a sense of ownership of the content and a sense of belonging in the mathematics department, leading to greater teacher retention.
- Recognize and support the need for elementary teachers to receive math-specific support and mentoring (see the "Content Focused" section below).
- Ensure that beginning mathematics teachers have comparable access to mathematics teaching resources (including technology, teaching spaces, and materials for hands-on instruction) as other mathematics teachers in the school.

- Involve new teachers in available professional learning communities, lesson study, or the like, particularly math-specific ones, to promote and aid regular reflection on their practice (Fulton and Britton, 2010).
- Encourage new teachers to attend mathematics teacher conferences, institutes, and workshops (and financially support them to do so).
- Ensure that beginning teachers understand who their students and families are, in particular their emerging multicultural learners, their interests, aspirations, and cultural and environmental backgrounds, and how to use those as resources for learning.

Ongoing Professional Learning for In-Service Teachers

A key component of professional learning, effective professional development*,* is vital to improving student learning outcomes. Effective professional development is structured professional learning that results in changes in teacher practices. Although there are many approaches to professional development—along with multiple aspects to each approach—some strategies and components have been shown to be more effective than others.

Characteristics of Effective Professional Development

In *Principles to Actions* (2014), NCTM connects education research to teaching practice with professional learning materials to help educators learn specific, research-based teaching practices. Moreover, the Learning Policy Institute's (LPI's) review of 35 rigorous studies on the implementation of professional development for teachers noted several elements of effective professional development that ultimately improve student outcomes (Darling-Hammond, Hyler, and Gardner, 2017). The LPI review found that generally, effective professional development is content focused, based in active learning, includes collaboration, uses instructional examples, provides coaching and expert support, includes feedback and reflection, and has a sustained duration. These characteristics are further described as follows.

Content Focused

Professional development in any discipline has been found to be most effective when the content knowledge in that area—in this case, mathematics—is a primary focus. Teachers must have opportunities to explore mathematical big ideas through rich, authentic, culturally relevant tasks to both deepen their own understanding of mathematics and better anticipate the challenges students might encounter and the strategies they may rely on to respond to them. These big ideas include the mathematical practices as central aspects of mathematics, equal in import to content standards. Professional development that introduces perspectives or teaching approaches without intentional connections to mathematics is unlikely to bring about much change in teachers' practice. Professional development that blends pedagogical and learning knowledge with mathematics knowledge has much more potential to result in powerful changes in students' learning experiences than that which focuses on pedagogy or content knowledge separately.

Many teachers have experienced mathematics as a set of procedures to be memorized. This narrow understanding makes access to opportunities to experience mathematics differently themselves all the more important, lest their own students have their mathematics identities shaped by similarly limited experiences of mathematics. As described in chapter 1, the goal is that students achieve conceptual understanding, problem-solving capacity and procedural fluency (in the full sense of the word fluency introduced in chapter 1) in mathematics. When teachers work on rich, authentic, culturally relevant mathematics tasks—through which they can ask their own questions, reason and communicate with others, and develop curiosity and wonder—they start to see mathematical connections they may never have seen before. This often changes teachers' relationships with mathematics, which is an important precursor to changing their teaching (see also Anderson, Boaler, and Dieckmann, 2018). This experience takes time and needs to be carefully organized, with teachers working together on mathematics in a supportive environment with an expert facilitator. Face-to-face professional development is the ideal way to encourage this experience, but online courses can also provide this experience, especially when teachers receive funded time to take the courses in groups.

Based in Active Learning

Teachers benefit most from professional development that engages them in the process of actively designing and trying teaching strategies and that provides them with opportunities to engage in the same style of learning they are designing for their students. Such professional practice relies on authentic artifacts, interactive activities, and other strategies to provide deeply embedded, highly contextualized professional learning. This approach moves away from traditional learning models and environments that are lecture based and fail to connect to teachers' classrooms and students. Instead, teachers should have opportunities to make sense of student thinking (in order to assess students' funds of knowledge and other assets—such as reasoning and communication practices—that will help drive teacher actions), reflect on their own and one another's instructional practices, and discuss connections to their own classroom. Classroom video is a powerful resource for such reflections and discussions. For example, professional development may include opportunities to watch videos showing linguistically and culturally diverse communities of English learners working to high levels with an expert teacher. Videos and other records of practice such as student work should be at the center of professional development opportunities.

Includes Collaboration

Effective professional development requires time and resources for teachers to share ideas and collaborate in their learning, often at the school level. Working collaboratively allows teachers to create professional learning communities that can positively change the culture and instruction at a classroom, grade, department, school, or district level. As teachers work together on mathematics instruction, they experience the collaborative, connected mathematics experience as a template for their own classrooms. They can also share experiences, including challenges, successes, and insights, to support one another in planning and implementing lessons. Professional learning communities are also important places where teachers can consider ways in which mathematics instruction can recognize students' cultural and linguistic assets and ways to draw on those assets to make contexts and problems ever-more authentic.

Uses Instructional Examples

Seeing lessons, tasks, and curriculum in action is a powerful tool for providing teachers with opportunities to experience best practices firsthand. Teachers may view examples that include lesson plans, unit plans, sample student work, observations of peer teachers, and video or written cases of teaching, such as the many vignettes presented in this framework. Teachers benefit from opportunities to discuss examples of teaching, reflect on current practices, and make connections to their own classrooms.

Effective professional learning must build teachers' capacities to notice, analyze, and respond to students' thinking (NCTM, 2014, 101). Professional learning built around artifacts of practice such as student work (written, video, or other) provides time and support to develop these capacities.

Provides Coaching and Expert Support

Implementing new teaching approaches can create challenging transitions in particular classrooms, schools, or even districts. Fortunately, coaching and expert support—especially from district and county mathematics coaches—have proven extremely effective in responding to these challenges when such support is structured around a particular purpose (e.g., for adopting new curricula or implementing specific new instructional practices) and is aligned with school-wide goals and priorities. Well-trained peers and teacher leaders with expertise in particular approaches can be powerful facilitators of growth by encouraging, modeling, and sharing insights—particularly when supported by the administration and by appropriate structures. These leaders can spend time observing teachers' instructional practices, recognize assets that teachers can build on, and work with teachers to develop the capacity to implement rich, student-centered mathematics lessons.

Includes Feedback and Reflection

High-quality professional development ensures that teachers are afforded dedicated time to think about, receive input on, and make changes to their practice. Reflection and feedback enable teachers to establish and refine realistic goals for changing their practice as they move toward expert visions of practice.

Formative assessment that provides evidence of student learning on rich assessment tasks provides one source of feedback that can be generative when combined with opportunities for collaborative teacher learning. For example, Boaler and Foster (2021) describe achievement gains that resulted when teachers in a group of districts participated in collaborative professional development focused on helping them teach broader and deeper mathematics to a wide range of students. The professional development engaged teachers in a formative assessment cycle (Briars et al., 2013; Foster and Poppers, 2011) centered around rich mathematical tasks designed by the Mathematics Assessment Resource Service (MARS). This process helped them to analyze and reflect on their students' learning while gaining knowledge of mathematics content and pedagogy, which enabled them to craft and teach new lessons that successfully engaged their students in key mathematical concepts.

Has a Sustained Duration

Effective professional development provides teachers with adequate time to learn, practice, implement, and reflect on new strategies that facilitate growth in their practice. Professional development that engages teachers in making incremental changes over time (and reinforces existing effective practices) can bring about lasting positive change.

Planning for Effective Professional Learning

Achieving this framework's vision of mathematics education will require improved systems of professional learning. Teachers, specialists, paraprofessionals, and school and district leaders should articulate personal and collaborative learning goals across grade levels and departments, focusing on curriculum, instruction, and assessment strategies that embrace the vision of the CA CCSSM and this framework. Schools, districts, and other local education agencies (LEAs) must become “learning organizations” (Senge, 1990) engaged in continuous improvement around the teaching and learning of mathematics. At every level (grade, department, school, and district) educators must share a vision that focuses on student learning, collaboration, collective inquiry, shared practices, reflection, and results (DuFour, 2004; Hord and Sommers, 2008; Louis, Kruse, and Marks, 1996). As discussed in the “Role of Parents, Guardians, and Families” section later in this chapter, families are collaborators in this shared vision. Families’ involvement provides educators and administrators with a better, more holistic understanding of students’ learning needs.

County offices of education, districts, schools, and other LEAs providing professional learning can use the report “Effective Teacher Professional Development” (Darling-Hammond, Hyler, and Gardner, 2017) as a resource for planning these types of learning experiences. This report provides much more detail about the features of effective professional learning described above.

Another resource for those designing professional learning opportunities is the *Professional Development Design Framework* (Loucks-Horsley et al., 2010). Through their research with national professional developers, Loucks-Horsley and her colleagues found that effective programs had several common characteristics: They were designed to meet various factors, to change over time, and to adapt to particular goals and contexts. They did not rely on formulas; instead, the designers used a process of thoughtful, conscious decision making. The authors used these factors and processes to create the framework shown in figure 10.4.

Figure 10.4 Professional Development Design Framework

Source: Loucks-Horsley et al., 2010.

At the center of the design framework, illustrated in the six squares connected with horizontal arrows, is a planning sequence that includes the following topics: (1) committing to a vision and a set of standards; (2) analyzing student learning and other data; (3) setting goals; (4) planning; (5) doing; and (6) evaluating results. The circles above and below the planning sequence represent important inputs into the design process that can help designers of professional learning make informed decisions. These inputs prompt designers to: consider the extensive knowledge bases (knowledge and beliefs) that can inform their work; understand the unique features of their context; draw on a wide repertoire of professional development strategies; and wrestle with critical issues that instructional reformers will encounter.

While there is no exact starting place for using the design illustrated in figure 10.4, effective planning should avoid starting with strategies—though they may seem most appealing. Instead, the use of evidence (derived through questions such as, What are the assets? or, What are the needs?) is encouraged. Designers should think about short- and long-term approaches (up to five years) as well as teacher career trajectories and plan to support teachers accordingly (Task Force on Educator Excellence, 2012).

However, those developing professional learning must also remain mindful of the need to stay flexible and adaptive. They should include openness to refining their ideas as they evaluate the implementation process. As the design and implementation phases are taking place, recommendations from *Innovate: A Blueprint for Science, Technology, Engineering, and Mathematics in California Public Education* (STEM Task Force, 2014) and the characteristics of effective professional learning should also be considered.

Note that although the framework in figure 10.4 is arranged as a linear and sequential model, it need not be employed as such. What is most important is to pay attention to the four core design inputs, where they impact the design of the program, and how they are addressed during implementation.

Models and Strategies: Effective Professional Learning

The characteristics of effective professional learning can be implemented through many professional development models and strategies, including the following:

Professional Development Models

- Professional Learning Communities (PLCs): PLCs provide opportunities for teachers to collaborate with each other and for administrators to collaborate with teachers in a team setting.
- Communities of Practice: Communities of practice are "...groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly" (Wenger-Trayner and Wenger-Trayner, 2015). In educational settings, PLCs are often site-based, whereas communities of practice often connect educators across sites, helping provide additional contacts and resources for improving practice.
- Classroom Coaching: A mathematics coach is an individual who is well-versed in mathematics content and pedagogy and who works directly with classroom teachers to improve student learning of mathematics (Hull, Balka, and Miles, 2009).
- Lesson Study: See below.

- Mathematics Labs: Mathematics labs provide a collaborative design and instruction cycle, similar to Lesson Study but with collaborative instructional decisions even during the lesson's implementation (Kazemi et al., 2018).
- Content-Intensive Institutes with Follow-Up Workshops: See below.

Professional Development Strategies

- Backward Design: Backward design focuses on the importance of student learning outcomes in lesson design.
- Universal Design for Learning (UDL): This strategy focuses on the implementation of and alignment with the guidelines of UDL.
- Networking and Community Building: These strategies focus on building a community around mathematics instruction.
- Partnerships: Partnerships with university mathematics and mathematics education faculty help bridge the research–practice divide.

Three models that are supported by research into effective professional development in mathematics are explored below. The first, lesson study, offers sustained content-focused courses with school-year follow-up and coaching. In a survey of the effectiveness of 643 professional development models, only two models were found to have a significant positive effect on students' learning—lesson study and sustained content-focused summer courses with pedagogy-oriented structured academic year follow-up (Gersten et al., 2014). Coaching models are very common in California schools, but "...there is little empirical evidence that coaching improves teacher practice" (Desimone and Pak, 2017). However, some structured coaching models show more promise for instructional improvement than individual one-on-one models (Gibbons, 2017).

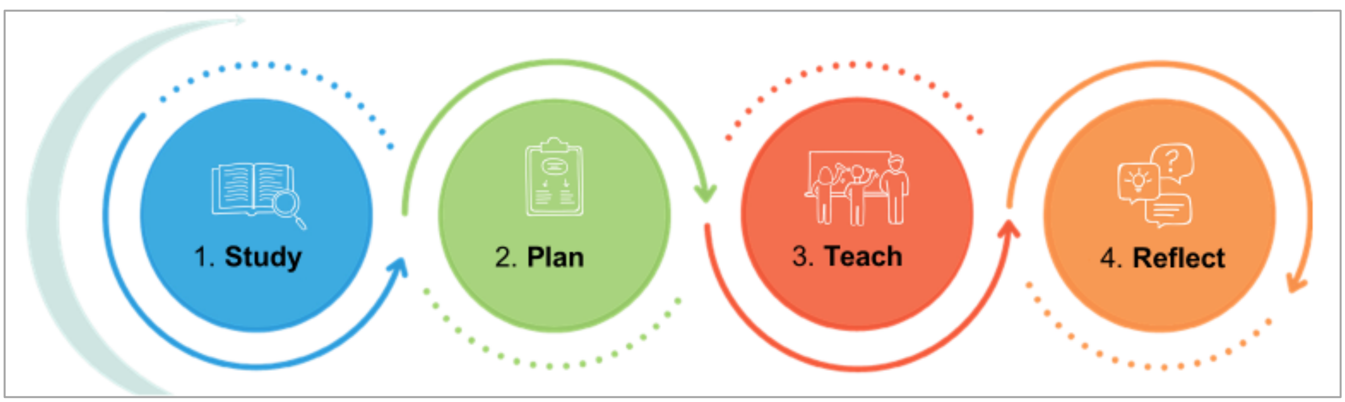
Lesson Study

Lesson study is a type of professional learning where teachers engage in an inquiry cycle that supports their ability to experiment, observe, and improve their teaching by collaboratively researching, creating, teaching/observing, and revising a lesson. Lesson study, which originated in Japan, has been shown to be an effective model for professional development with its deliberate focus on planning and teaching practice as well as inquiry, creativity, and collaboration (Lewis and Hurd, 2011).

The proven effectiveness on student learning led the California Mathematics Project (CMP), one of the nine subject disciplines that comprise the California Subject Matter Project, to formally adopt lesson study as a preferred means of professional development in 2018. CMP later spearheaded the creation of the California Action Network for Mathematics Excellence and Equity (CANMEE, n.d.), which supports California schools and districts in implementing high-quality lesson study. The Lesson Study Group at Mills College provides many online resources to support such implementation.

The lesson study cycle consists of four phases (Mills College. n.d.), as shown in figure 10.5.

Figure 10.5 The Four Phases of the Lesson Study Cycle



Source: Mills College. n.d.

In the Study phase, a team of teachers collaborates to:

- Identify long-term goals for students.
- Choose the subject and unit to investigate.
- Study standards, research, and curricula.

In the Plan phase, using insights from the Study phase, the team:

- Examines the unit and chooses one lesson to plan in depth.
- Articulates the lesson goals.
- Tries the lesson task and anticipates student thinking.
- Identifies data to be collected during the lesson.

In the Teach phase, the team puts that lesson into action:

- One team member teaches the lesson.
- Other team members observe and record student thinking and learning.

In the Reflect phase, the team then reflects on their work by:

- Meeting after the lesson to discuss data on student thinking and learning.
- Having an outside specialist provide further commentary.
- Reflecting on what they learned during the cycle as a whole.

Some or all of these phases are often repeated by a team, since a team often wishes to redesign a lesson based on realizations made in the Reflect phase and teach it again to another class of students.

It is important to note that the “product” of a lesson study cycle is more than a refined lesson plan. Team members deepen their understanding of content and student thinking, their commitment to collaboration, and their ability and inclination to base instructional decisions on evidence of their students’ thinking.

Lesson study is particularly fruitful when teachers have access to research-based mathematics resources (Lewis and Perry, 2017; Perry et al., 2009). Lewis and Perry (2017) found that locally led lesson study teams randomly selected to receive fraction resource kits produced significantly greater increases in students’ and teachers’ fractions knowledge than teams that did not receive the resources. The resource kits included research articles, video, student work, and research-based curricula with a focus on linear representations of fractions such as number lines (LSGAMC, 2022). The snapshot below illustrates lesson study being used at the second-grade level.

Second-Grade Snapshot: Lesson Study

Equity focus: Linguistically and culturally diverse English learners' productive language use in mathematics

Source: The California Action Network for Mathematics Excellence and Equity (CANMEE) Steering Committee, adapted

The second-grade teachers at 54th Street Elementary met during their professional learning community time to discuss the performance of their emerging multicultural learners in mathematics. Each teacher noticed that their English learners were having difficulty explaining their solutions to mathematics problems orally and in writing. They invited the English language development (ELD) specialist to the meeting to hear their concerns and obtain suggestions for addressing the students' needs.

The ELD specialist had recently observed a lesson at another elementary school focused on equity. The ELD specialist suggested that the second-grade teachers consider participating in a lesson study focused on building the agency of their multilingual students. The teachers decided to engage in a lesson study cycle of 30 hours and followed the lesson study model of Study, Plan, Do/Test (Teach), and Reflect.

As part of the equity focus of the CANMEE lesson study process, each teacher selected four designated English learners as focal students from their classes and interviewed them to determine their strengths and challenges in mathematics. Based on the content of interviews and classroom observations, the teachers drafted assets-based descriptions for each, then met and shared their focal student descriptions.

During the Study phase of their lesson study, the teachers read literature that centered on effective practices for English learners, such as the *English Language Arts/English Language Development Framework* (CDE, 2014), the *English Learner Roadmap* (CDE, 2017), and important research (Moschkovich, 2012; Ramirez and Celedón-Pattichis 2012). As part of the Plan phase, teachers designed a mathematics lesson with a task that required students to record their thinking in a journal and share their ideas with a partner. One of the goals for the focal students was to increase their productive language skills. The teachers engaged in the mathematics task themselves to anticipate both productive and unproductive student strategies. The teachers developed questions to ask those students who used unproductive strategies and consulted with the ELD specialist for additional resources. The specialist posed questions to allow the teachers to do the thinking.

In the Do/Test (Teach) phase, one of the teachers on the team volunteered to teach the lesson while the other teachers observed the focal students during the lesson to determine the effect of the lesson they designed. An outside expert in mathematics content was invited to provide feedback on the mathematics content of the lesson, serving as the mathematics commentator. The ELD specialist served as the equity commentator. The ELD specialist observed the focal students' interaction with the lesson and peers as well as their productive language skills—in particular, aspects of the lesson design that seemed to facilitate productive language opportunities. The second-grade teachers also invited other educational partners, including colleagues at the school and parents, to observe the public lesson.

After the lesson was taught, as part of the Reflect phase, the team of teachers shared their thoughts and observations about the impact of the collaboratively planned lesson on the participation and learning of the focal students. They also identified ways to improve their teaching practice moving forward. The mathematics and equity commentators shared their observations of the lesson and provided suggestions for next steps. Other observers (including parents) also made comments about the lesson.

At the end of the cycle, the second-grade teachers reflected on the professional learning experience. They noted the value in the ability to collaborate with their peers about a problem of practice that was specific to their school. The teachers also felt that the support from the ELD specialist was critical to their success. They all noticed an increase in agency among the focal students as a result of the lesson study process. Lastly, the second-grade teachers noted feeling more confident about their ability to meet the needs of their students who are emerging multicultural learners.

(*end snapshot)*

Content-Focused Workshops with Follow-Up

"One and done" professional development sessions have shown little impact on teaching practice or student learning (Darling-Hammond, Hyler, and Gardner, 2017). In addition to lesson study, sustained content-focused professional courses/workshops with school-year pedagogy-focused follow-up have demonstrated positive impact on student learning (Gersten et al., 2014). Several partner organizations in California work with districts and schools to provide these opportunities.

Structured Coaching

The central goal of mathematics coaching is to support mathematics teacher learning and do so embedded in the contexts in which mathematics teachers do their work. Coaches can engage individual teachers and groups of teachers in a variety of potentially productive activities (Gibbons and Cobb, 2017), such as co-planning, examining student work, modeling instruction, and side-by-side coaching. In each, the teacher and coach co-participate in some way in the work of teaching—e.g., preparing, enacting, or reflecting—and work together to make sense of mathematics content, student thinking, and pedagogy. For coaching to support teacher learning, teachers and coaches must make visible what they are noticing (Sherin et al., 2011), how they interpret what they see, and how and why they are making pedagogical decisions (Horn, 2005; Loughran, 2019).

Instructional coaching best contributes to school-wide mathematics instructional improvement when it is used as a tool to support the collective learning of teachers (Gibbons, 2017). In other words, the characteristic of effective professional learning that "provides coaching and expert support" does not stand alone; designating a "good mathematics teacher" as a coach has not proven to improve teaching practice by itself. Coaching is effective when it is structured to provide more than a model/co-teach/you teach feedback loop: "Coaches need to engage teachers in fundamental dialogue about mathematical content, mathematical learning, and student understanding" (Campbell and Griffin, 2017). Thus, coaching is effective when it is part of a broader professional learning plan that incorporates most or all of the other characteristics of effective professional learning, as described in the coaching vignettes *Making Sense of Content, Student Thinking, and Pedagogy*. In each of these vignettes, the teachers' goals for professional learning shaped both *what* the teacher and coach worked to make sense of—content, student thinking, or pedagogy—and *how* they worked together. Effective coaching aligns the teachers' goals with coaching activities that allow the teacher to actively make sense with a knowledgeable colleague.

Building Teacher Leadership

Ultimately, successful development and implementation of effective professional learning for teachers relies on expertise, which requires district capacity. Using in-house personnel who may lack the necessary expertise is not effective for creating lasting, meaningful changes that students are entitled to receive. Yet the use of outside expertise can, over time, diminish the district's capacity to build internal leadership. Districts must consider ways to build teacher, curricular, and administrative leadership, with the assistance of outside sources, to strengthen their long-term capacity to improve mathematics learning. Every district will have some teachers who actively seek opportunities to develop personal capacity to provide authentic mathematics learning opportunities. Identifying these early adopters and supporting their learning—as well as developing their leadership in supporting other teachers—can be an effective way to strengthen a school or district's professional learning networks for mathematics.

This section begins with the development of teacher leadership as a core strategy for supporting improvement in teaching and learning. Research indicates that turning professional learning experiences into changes in teaching and learning practices requires leadership and support (Lieberman and Miller, 2008; Weiss and Pasley, 2009). Teacher leadership is associated with increased teacher learning and the creation of collaborative professional cultures (York-Barr and Duke, 2004; Werner and Campbell, 2017). It is also positively related to increased student achievement (Waters, Marzano, and McNulty, 2003).

As Julian Weissglass (1998) states, “Teacher leadership is about taking responsibility for what matters to you.” Everyone has the capacity for leadership, and one goal of mathematics teacher leadership is to have many, rather than a few, people leading creatively every day and in all aspects of their lives (Kaser et al., 2013). In other words, teachers in multiple roles are leaders, ranging from those seeking to be or designated as teacher leaders to department chairs, teachers on special assignment, mentors and coaches. This view of teacher leadership differs from the traditional view in that leadership is not about power and authority. Instead, it embraces five practices of exemplary leaders (Kouzes and Posner, 2003), as listed in Figure 10.6.

Figure 10.6 Practices of Exemplary Leadership

Practices of Exemplary Leaders	Descriptor
Challenging the process	Searching for opportunities to change the status quo and innovative ways to improve
Inspiring a shared vision	Seeing the future and helping others create an ideal image of what the organization can become
Enabling others to act	Fostering collaboration and actively involving others
Modeling the way	Creating standards of excellence and leading by example
Encouraging the heart	Recognizing the many contributions that individuals make, sharing in the reward of their efforts, and celebrating accomplishments

Source: Kouzes and Posner, 2003

Leadership development requires explicit attention, clear expectations, and resources, time, and expertise (Hopkins et al., 2013; Yow and Lotter, 2016). Mathematics teacher leaders need to continually build their (1) in-depth understanding of the mathematics content and practices of the CA CCSSM; (2) thorough knowledge of the best practices in teaching and learning based in authentic contexts and problems; (3) understanding of school culture, organization, and politics; (4) understanding of change theory; (5) knowledge of how adults learn; and (6) practices that embrace continuous improvement. Additionally, leaders need skills that include facilitation and communication, data use, decision making, and organization.

Teacher leaders can take on a variety of roles to help colleagues and other educators, as well as parents, guardians, and community members, become more aware of and aligned with improvements in mathematics teaching and learning. These roles include leading in the areas of (1) instruction and assessment; (2) curriculum and instructional materials; (3) school culture that is supportive and proactive for the implementation of the CA CCSSM; (4) community support and advocacy for active, authentic mathematics instruction; and (5) mathematics classroom implementation of the California ELA and ELD Standards. An explicit current in all of these roles must be access and equity for all students.

To develop needed knowledge and skill sets, teacher leaders need professional learning targeted toward leadership. Learning experiences are most productive when they occur over time, provide feedback, are anchored in the practice of instructional leadership, and ground the leaders in mathematics practices and content (Darling-Hammond, Hyler, and Gardner, 2017; Fullan, 2015; Kaser et al., 2013). Districts need to develop leadership programs that embrace these attributes and/or encourage teacher leaders to participate in leadership experiences through programs such as the California Mathematics Project.

Teacher leadership can manifest in many forms, including presenting (at the school site, district, or professional organization level), consulting (as informal specialists for other mathematics teachers), facilitating (through site-level department collaboration, lesson study groups, or district-level efforts such as assessment and vertical alignment choices), and coaching.

The extensive literature on teacher leadership cited in this section provides additional sources for further learning by those seeking to empower and support teacher leaders.

Governance and Administrative Leadership for Professional Learning

School boards, working within their responsibilities, play an important role in supporting administrators and teachers to increase instructional knowledge and skills. When the board aligns its governance responsibilities and focuses on goals to increase students' mathematical understanding and success, district structures and resources strengthen administrative leadership.

Administrators play a key role in helping create and sustain a multilayered system of support for teachers in their pedagogy and professional learning. There are several dimensions to the types of specific support administrators can provide, including having well-informed conversations with teachers about instruction and assessment and giving teachers feedback on instruction.

Together with their teaching staff and paraeducators, administrators may need to seek opportunities to understand more about the nature of mathematics learning and teaching presented in this framework. Leadership beliefs regarding mathematics instruction should be reconsidered. For example, maintaining beliefs such as "fidelity to the curriculum" can undermine the focus and coherence called for in chapter 1. It is critical that clarity about focus, coherence, and rigor in mathematics be communicated at district, school, and department levels. Addressing policies and practices around course offerings, placement, and de-tracking are essential conversations to be had at all levels.

Unlike teachers, administrators are in a unique position to support and enact changes on a program level, rather than focus solely on the classroom. Administrators should provide support for discussions on district- and school-wide changes in practices and on policies that can result in more equitable mathematics learning outcomes for all students. In establishing and maintaining regular communication with teachers about their teaching, their students, and the curriculum, administrators play a pivotal role in instilling the confidence and vision necessary to help teachers explore new ways of ensuring all students can engage with mathematics. The guidance presented in this framework can serve as a starting point in helping to structure these conversations.

Administrators should be aware of this framework’s responses to the challenge posed by the principle of coherence. The big ideas of mathematics unfold in progressions across grades (thus, grade-band chapters rather than individual grade chapters), and are taught by way of intriguing investigations with relevance to students’ lives. The learning progressions chapters (chapters 3, 4, and 5) highlight the value in building powerful ideas about numbers and data whose meaning grows clearer over time and resonates with each subsequent grades’ topics. Learning is focused on building productive habits of mind such as exploration, discovery, and communication involving mathematics.

Administrators should also be aware of the general principles guiding the development of the grade-band chapters (chapters 6, 7, and 8). These include designing lessons from a small number of big ideas in each grade band; spending a preponderance of student time on authentic problems that engage multiple content and practice standards situated within one or more big ideas; focusing on connections, to students’ lives and among mathematical ideas; and using teaching strategies that show connections between different mathematical ideas on various topics across grade levels.

Working with their teaching staff, administrators may need to identify opportunities to learn more about inclusive teaching strategies. Chapter 2 sets out the important qualities of mathematics classrooms that encourage student engagement and equitable outcomes. Through professional workshops, conferences, or other professional learning, administrators can support teachers in learning to use engaging, equitable strategies. Partnerships with parents, families, and caregivers can also provide valuable opportunities for administrators as they work with teachers in addressing the totality of students’ learning experiences. Family partnerships and experiences, especially given families’ cultural and linguistical diversity, can create rich avenues for professional learning for teachers and teacher leaders. Administrators should also draw on teacher leaders at their school site or within their district who can provide support and knowledge of inclusive teaching approaches, especially those that focus on cultural and linguistic diversity and on students with learning differences.

An important idea conveyed in this framework is that all students deserve access to a high-level mathematics curriculum. The chapter on data science (chapter 5) discusses ideas that will be new to many administrators about how to support students' learning of statistics and data science from elementary through high school. Classroom use of real data relevant to students' lives, and the encouragement of students to ask questions it raises in their minds has the potential to broaden STEM participation and make mathematical learning more equitable. Holding equity as a guiding principle and working to encourage equitable participation in new courses is paramount for administrators as new courses are developed and introduced.

Administrators are urged to read all of chapter 9 as they engage in conversations with teachers, school boards, and parents on the ramifications of acceleration and tracking. They need to work with these same groups to carefully consider the many alternatives to tracking that afford better access to higher-level mathematics for all learners, as discussed in chapter 9.

The instructional vignettes in the framework can help administrators develop an awareness of the different teaching strategies and classroom conversations that provide opportunities to improve professional practice. Such vignettes also encourage administrators to reflect on the ways they can nurture these types of experiences for their mathematics teachers. The vignettes highlight the central role of classroom discourse and rich, open tasks in teaching and learning mathematics.

One key perspective for administrators to recognize is that standards-driven instruction does not mean that each task results in the learning of a single standard. In fact, multiple standards can often be learned through engagement with rich tasks with multiple access points as called for in chapter 2. And mastery-based assessment at the "big idea" level, as described in chapter 12, helps to reinforce the experience of mathematics as a sense-making, relevant activity. Administrators who understand that exploring a big idea through a single, rich task provides opportunities for students to communicate their thinking with their peers and their teacher also understand that this approach to instruction often results in the learning of multiple standards—and does so in ways that foster both a positive disposition toward mathematics and learning that lasts.

Additionally, administrators must acknowledge the inequities often perpetuated through traditional assessment strategies in the mathematics classroom and how these assessment approaches can be re-envisioned (as described in chapter 12) to provide a balanced approach in assessing the effectiveness of mathematics instruction. Administrators should look critically at program data to determine where their districts need to focus more attention to ensure equitable access to mathematics throughout the grades and enable students from all backgrounds to succeed. Transcript analysis and course-taking patterns, correlated with metrics of achievement, provide a broader view of student success than solely focusing on exam achievement. The results of multiple assessment strategies—rather than a single score on a test—reflect a more complete understanding of student learning. Standards-based assessment provides an approach to grading that focuses learning on standards and mastery rather than emphasizing grade ranges or percentages. Broadened approaches to assessment in a district/school often mean that administrators prioritize participation in ongoing professional learning on the topic of mathematics education and assessment of learning. Administrators can leverage their understanding and use of the Multi-Tiered System of Support (MTSS, CDE, n.d.) by supporting teachers in aspects of MTSS implementation, such as integration of instruction with intervention and a focus on continuous improvement.

Several ways that administrators can help support and incentivize effective professional learning are outlined in "Effective Teacher Professional Development" (Darling-Hammond, Hyler, and Gardner, 2017):

1. Since a critical component of rich learning is the planning time and pedagogical knowledge necessary to facilitate an active mathematics learning environment, administrators should prioritize time for professional learning and collaboration when designing schedules. Professional learning communities, peer coaching

and observations across classrooms, and collaborative planning all provide important opportunities for educator learning.

2. Periodic needs assessments (at the school or district level) use staff surveys to identify areas of professional learning that educators desire and need most. Such routines help ensure that professional learning is connected to practice and makes an impact on practice much more likely.
3. District and school administrators should identify and develop expert teachers as mentors and coaches to support the professional learning of other educators. These expert teachers need their own support, structure, and professional learning in order to be effective.
4. Districts and schools should ensure that professional learning opportunities are integrated with efforts to implement legal requirements, such as the Every Student Succeeds Act (ESSA) school improvement initiatives. Mandates, such as the use of data to inform instruction and the creation of positive and inclusive learning environments, tend only to be effective when educators experience them as supportive of their efforts to improve classroom practice, as opposed to compliance exercises that add more paperwork to busy days.
5. To address professional learning needs of rural communities and to develop intra-district and intra-school collaboration, Titles II and IV of ESSA should be used to support technology-facilitated opportunities for professional learning and coaching.
6. District and school administrators can seek funding that supports professional learning opportunities and ensure that these opportunities earn continuing education units. These opportunities can include many of the types listed below, such as institutes, workshops, mathematics-specific conferences, and seminars, and also sustained engagement in collaboration, mentoring, and coaching. Possible funding sources include Local Control Accountability Plans, state and federal grant programs, community/business partnerships, and foundations.

Some specific resources to aid instructional leaders in supporting quality mathematics instruction include organizations that are available to partner with schools, as well as observation and planning guides. These organizations and tools enable administrators to convey high expectations for mathematics instruction—expectations made attainable by providing teachers with resources, including time for planning lessons, professional learning, and collaboration. These expectations focus on and align with agreed-upon school-wide priorities and strategies. As teachers implement their plans, administrators can provide constructive, informative feedback that builds on teachers' strengths. In collaborating with teachers around lessons, administrators can engage teachers in frequent, productive conversations about mathematics teaching and provide relevant feedback on instructional practices.

By contrast, the general pattern in many California schools is for a classroom teacher to be observed formally once a year—a practice that is insufficient for administrators to gain an understanding of teachers' instruction and insights on how to support it. Instead, scheduling frequent and sustained interaction with teachers improves administrators' engagement with students and teachers. Routine interaction allows administrators to glean a more complete picture of the instructional practices used by their teachers and determine the kind of support that would bring about positive growth.

Role of Parents, Guardians, and Families

While the school classroom is a primary learning environment for mathematics education, home and community also play significant roles. Through involvement at every level, parents, guardians, and families can motivate students to develop a lifelong appreciation of mathematics learning. Families can also provide a supportive home setting for students to learn and prepare for school. Partnering with parents, guardians, and families in understanding and supporting authentic mathematics education and active learning pedagogy is key.

A substantial body of research asserts that "effective family engagement depends on the close working relationships between teachers and each child's family (Niebuhr, Arseo, and Simeon, 2021) and that these relationships require building capacity among families and educators. As happened during the global pandemic of 2020–21, families can support learning as "co-creators, supporters, encouragers, monitors, advocates, and models" (Mapp and Bergman, 2019). Families are key in supporting the development of future mathematicians by increasing students' confidence, developing a growth mindset, providing examples of math applied to real-life situations, and providing out-of-school activities. Creating a bridge between children and their families helps children to deepen their connection to their learning and be more successful academically.

The passage below from *Black, Indigenous, and Latinx Parents as Partners in Mathematics Education* by TODOS: Mathematics for ALL (2020) provides insights about the assets parents bring when invited into the teaching and learning process:

> Black, Indigenous, and Latinx parents have a lot to offer classrooms. However, they are not always asked to join and be a part of the instruction. Ishimaru, Barajas-López, and Bang (2105) have argued for the involvement of parents from nondominant groups in schooling, not as passive recipients of knowledge but as "expert collaborators and fellow leaders." (p. 14). Given our current expectation of online and hybrid classes, schools can develop an online learning culture leveraging school/home connections that support mathematics identity and agency for students and parents. Research on Latinx parents visiting classrooms suggests that observations and debriefs of classroom visits were one way that parents were able to both reflect on ways to support their students and develop leadership in mathematics education (Civil and Menéndez, 2012).

Because the CA CCSSM and this framework present mathematics instruction that is significantly different from what many parents experienced as students, it is critical to educate parents and guardians about what to expect and about the reasons and research behind the changes. Educating and engaging parents and guardians should include opportunities for them to experience rich, authentic, culturally sustaining mathematical tasks in active-learning ways (including support for parents who speak languages other than English), not simply written descriptions of it. Validating and valuing parents', guardians', and families' central contributions to education is enhanced when they have opportunities to use their own language, culture, and knowledge through relevant experiences rooted in the school context.

Furthermore, parents and guardians who become more knowledgeable through such experiences can more effectively support students’ learning beyond the classroom. Parents and guardians can monitor their student’s progress not just for content knowledge, but for understanding of and engagement in mathematical practices or a developing inclination to use mathematics to make sense of their world. Parents and guardians can also foster social interactions (e.g., by providing support for collaborative classroom or out-of-classroom projects) and become involved in educational activities promoted at the school site (e.g., math fairs and math clubs). Finally, in addition to coordinating social events, parents may advocate for their children’s appropriate class placement with school academic counselors. Often this happens as students transition to middle and high school.

A model to support the development of family and school partnerships is the National Parent Teacher Association (PTA), which has developed standards for Family-School Partnerships. These standards focus on several aspects of the partnership, providing recommendations on how to foster trust and effective communication to support student success. In addition to the standards, the National PTA has developed a guide that provides a rubric with examples for what family–school partnerships look like at the emerging, progressing, and excelling levels. Parents, guardians, families, and school leaders may want to use these examples to evaluate and enhance the family–school collaboration at their school. Specifically, involving parents who have a background in mathematics (including in such areas as the building trades and cooking, as well as more traditional STEM areas) will help develop partnerships with the community that can provide much-needed support for classroom instruction.

Acknowledging cultural differences can further help educators include families as allies. It also provides educators with the opportunity to reinforce the importance of big picture ideas and to diminish formulaic thinking. For example, parents and guardians whose own schooling was in another country might possess different ways of solving mathematical problems. They may be hesitant to get involved in their student's mathematical learning for fear of teaching them "incorrectly" or differently from the approaches used in their student's classroom.

The California *ELA/ELD Framework* provides specific suggestions for parent, guardian, and family involvement when those families speak a language other than English or are new to the United States. For example, parents who have experience with mathematics and speak a home language that students also speak can provide welcome and beneficial support for the parents of those students who are not as experienced with mathematics (CDE, 2014, Chapter 11).

Collaboration Among Partners and Communities

As the sections above explain, a universe of partners and communities is involved in mathematics education, and they need to be aligned and work collaboratively. Teachers perform incredibly complex work that relies on thousands of instructional decisions every day (Ball, 2018). They need to understand their students' thinking, choose tasks, decide which questions to pose in discussion, select which (and whose) lines of inquiry to pursue with the class, and ensure that tasks and context are authentic and culturally relevant for all students. When educational partners and influencers outside of the classroom are not aligned—for example when a textbook does not align with the vision of classroom instruction—the work of teaching is made even more difficult, and improvement in instructional practice is impeded.

While implementation of the CA CCSSM has led to significant instructional change, the iterative nature of teaching means that improvement of mathematics teaching and learning is continuous. The many educational partners and communities whose efforts need to be aligned include (adapted from the *California Science Framework*, 2016):

- Teachers and teacher leaders prepared to engage in student-centered teaching that engages students in equity-oriented learning through authentic tasks and contexts that are relevant to those students based on their choices, interests, and aspirations

- School, district, and county office administrators who are knowledgeable and supportive of the changes demanded by the CA CCSSM and this framework
- Afterschool, early childhood, and other expanded learning opportunities aligned with and supportive of authentic mathematics learning that include collaborative and coherent efforts between teachers and other education support professionals
- College and university faculty involved in and advocating for high-quality mathematics instruction and preparation of future teachers
- Community members and parents, guardians, and families who understand the reasons for and are supportive of engaging in equitable approaches to mathematics teaching and learning
- Formal and informal learning environments, including museums, libraries, science centers, and other venues that are fully committed to supporting the CA CCSSM

Effective progress takes place when these partners and communities are aligned and work collaboratively within an ongoing cycle of implementation, reflection, and improvement of practice (Fixsen and Blase, 2009; Fixsen et al., 2005; Little, 2006; Penuel, Harris, and Debarger, 2015). The vision is for teachers and other educational partners to engage in a learning community that has the same characteristics—respect, commitment, intellectual engagement, and motivation toward continuous improvement—that all educators hope to create for students in California classrooms.

Ermeling and Gallimore (2013) present models of continuous improvement that have been embedded in school learning communities across 40 districts. These models focus on addressing learning needs common to community members; using analysis of evidence to drive planning and decision making; and critically questioning practices. To be effective, the learning community must operate in an environment of collaboration and trust among teachers and school leaders, each of whom recognize that improvement requires time, resources, continuous support, and an appreciation of risk-taking as new instructional approaches are implemented.

Improvement efforts in mathematics teaching and learning should focus on the sustainability of improved instructional practices and education programs as well as the sustainability of the professional learning cycle itself. This requires fostering a collaborative school culture that routinely engages educators, administrators, students, parents, guardians, families, education professionals, and community members (Fixsen and Blase, 2009). Such a culture allows all educational partners to understand themselves as advocates and supporters in the effort to improve students' experience and achievement in mathematics.

Finally, as discussed above, continuous improvement calls for teachers and educational leaders to examine personal beliefs and attitudes toward students and their families (CDE*,* 2014). Explicit reflection helps educators approach all students with a growth mindset disposition that both values the cultural resources and linguistic assets students bring to the mathematics classroom and supports them to use these resources while expanding and adding new perspectives and ways of appropriating and using mathematics. Teachers' beliefs about their students significantly affect those students' motivation, experience, and achievement (Heyder et al., 2020; Stipek et al., 2001).

Conclusion

A broad system of support to enable all students to succeed in mathematics learning consists of many interconnected parts. Teachers, as the drivers of learning, continually refine and adapt their practice to address the many dimensions of creating a rich mathematical learning environment focused on active learning for all students in their classrooms. By supporting teachers with the resources, time, insight, and encouragement to become ever-more effective practitioners of their craft, administrators serve a critical role in the system. The elements for effective professional development described in this chapter provide administrators and other stakeholders with guidance on creating high-quality learning experiences for teachers, and the examples listed are a small sampling of the variety of professional development experiences available. Supporting teachers, both in their own learning and in their teaching, ultimately supports the students who rely on these teachers.

Long descriptions for Chapter 10

Figure 10.1 The *Why, How,* and *What* of Learning Mathematics (accessible version)

Drivers of Investigation Why	Standards for Mathematical Practice How	Content Connections What
In order to… DI1. Make Sense of the World (Understand and Explain) DI2. Predict What Could Happen (Predict) DI3. Impact the Future (Affect)	Students will… SMP1. Make Sense of Problems and Persevere in Solving them SMP2. Reason Abstractly and Quantitatively SMP3. Construct Viable Arguments and Critique the Reasoning of Others SMP4. Model with Mathematics SMP5. Use Appropriate Tools Strategically SMP6. Attend to Precision SMP7. Look for and Make Use of Structure SMP8. Look for and Express Regularity in Repeated Reasoning	While… CC1. Reasoning with Data CC2. Exploring Changing Quantities CC3. Taking Wholes Apart, Putting Parts Together CC4. Discovering Shape and Space

Return to figure 10.1 graphic